SACRED GEOMETRY PRIMER
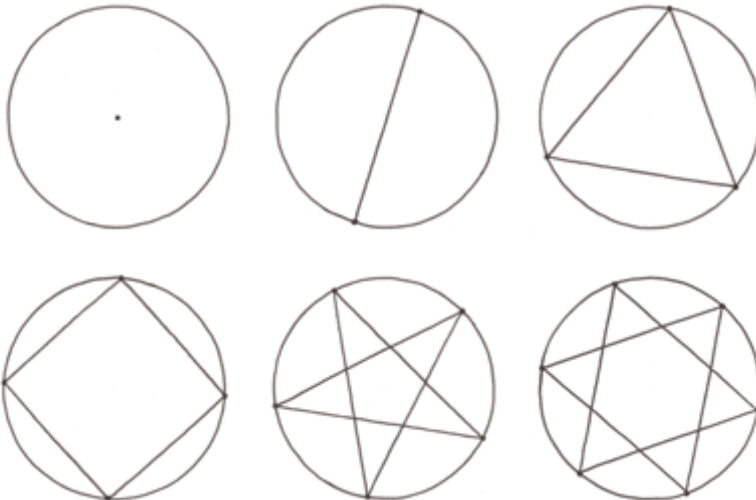
Publication in Class C

1–9

The numbers 1–9 tell the story of the creation of the material world.[1]

1

The 1 is the point in the circle; representative of Kether on the Tree-of-Life. The point at its center is that singularity in the whole (circle) of the Universe and the oneness from which the whole of creation emanates. The circle is the foundation from which all the other forms are built. Crowley notes in the Book of Thoth:

The Qabalists expanded this idea of Nothing, and got a second kind of Nothing which they called "Ain Soph"-"Without Limit". (This idea seems not unlike that of Space.) They then decided that in order to interpret this mere absence of any means of definition, it was necessary to postulate the Ain Soph Aur-"Limitless Light". By this they seem to have meant very much what the late Victorian men of science meant, or thought that they meant, by the Luminiferous Ether. (The Space-Time Continuum?) All this is evidently without form and void; these are abstract conditions, not positive ideas. The next step must be the idea of Position. One must formulate this thesis: If there is anything except Nothing, it must exist within this Boundless Light; within this Space; within this inconceivable Nothingness, which cannot exist as Nothing-ness, but has to be conceived of as a Nothingness composed of the annihilation of two imaginary opposites. Thus appears The Point, which has "neither parts nor magnitude, but only position".

But position does not mean anything at all unless there is something else, some other position with which it can be compared. One has to describe it. The only way to do this is to have another Point, and that means that one must invent the number Two, making possible The Line.

2

The 2 is the line; the connection of two points that divides the circle, as a cell splits in reproduction, and creation begins. Two is the duality that is the nature of the Universe wherein we dwell. Note that in the Qabalah, it is the second letter that is the foundation upon which the world is built.

Moving onto the next number, Crowley notes:

But this Line does not really mean very much, because there is yet no measure of length. The limit of knowledge at this stage is that there are two things, in order to be able to talk about them at all. But one cannot say that they are near each other, or that they are far apart; one can only say that they are distant. In order to discriminate between them at all, there must be a third thing. We must have another point. One must invent The Surface; one must invent The Triangle. In doing this, incidentally, appears the whole of Plane Geometry. One can now say, "A is nearer to B than A is to C".

3

The 3 is the triangle and as the first shape to create a surface, represents the movement away from one-dimensional reality to two-dimensional reality. Note the triangle represents the strongest structure in nature. Philosophically, it represents the dialectic; point, counter-point and conclusion.

4
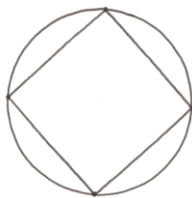
The 4 is the square, which offers support and stability through its even, balanced shape. It is the fourth point; representing the solid as a departure is made from two into three-dimensional reality. This is reflected in a four-fold nature (the space-time continuum) on the Earth with: 4 seasons, 4 directions, 4 phases of the Moon, 4 elements (Earth, Air, Fire, Water).

5

The 5 is the pentagram (an ancient symbol of regeneration and transformation) and a fifth principal; postulated as motion. It is also a symbol of the human form, with head, arms, and legs outstretched, welcoming experience. It is through our 5 senses that we navigate the material world.

6
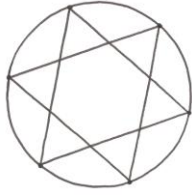
Movement implies consciousness; represented by the sixth Sefira (Tiphareth) on the Tree-of-Life. The 6 is the Star of David, two perfectly balanced, interlocking equilateral triangles. In Occult science, it gives us the axiom: 'As above, so below'; the uniting of spiritual plane with the material plane. It is the number of God (1) in man (5). Crowley states this clearly in his essay:

[The fifth point; motion] implies the idea of *Time*, for only through Motion, and in Time, can any event happen. Without this change and sequence, nothing can be the object of sense. (It is to be noticed that this No.5 is the number of the letter He' in the Hebrew alphabet. This is the letter traditionally consecrated to the Great Mother. It is the womb in which the Great Father, who is represented by the letter Yod which is pictorially the representation of an ultimate Point, moves and begets active existence).

There is now possible a concrete idea of the Point; and, at last it is a point which can be self-conscious, because it can have a Past, Present and Future. It is able to define itself in terms of the previous ideas. Here is the number Six, the centre of the system: self-conscious, capable of experience.

7

7 (Sat, the Essence of Being itself) is the most sacred of numbers; represented in cosmology by the seven sacred planets and reflected into the seven days of the week. The 7 relates to cycles of time and the movement of the sun and the planets as seen from Earth. Seven is also the sacred vibration found in the numeration of the chakras and the musical scale.

8
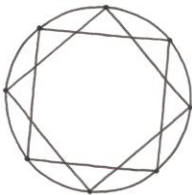
The 8 (Chit, Thought, or Intellection) is the octagon, the intermediate form between the circle (Heaven) and the square (Earth). As the 4 doubled, it is the number of ascension above the material world through spiritual perspective. The 8th step on the musical scale is the octave, which brings harmonic resonance and a sense of fulfillment.

9
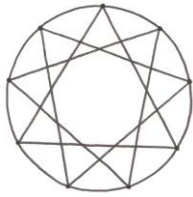
 The 9 (Ananda; translated as Bliss, being the pleasure experienced by Being in the course of events.) is the completion of the creative cycle; represented by the Astral Plane Magick. It brings attainment, compassion, and vision for the future through the wisdom gleaned from the journey through the other numbers. However the 9 is not an endpoint, but merely the closing of the circle, carrying us back to the 1 to begin the next evolution.
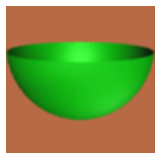
General Figures

The Sphere

While the sphere may be one of the simplest forms in sacred geometry, it is also the container that can hold all of the other forms. All measurements are equal in a sphere. It is a figure that is complete in its entirety. The earth, a seed, and an atom are all spheres.

 The sphere looks like the surface of a ball.

 The sphere has been cut in half to show that it is not solid.

 Bands of the sphere have been removed.

The Circle

A circle is another simple form found in sacred geometry. The circle is two dimensional and is a symbol of oneness. The ratio of the circumference of a circle to its diameter is called Pi. Pi is an irrational number and never ends nor does it ever repeat. It is infinite.
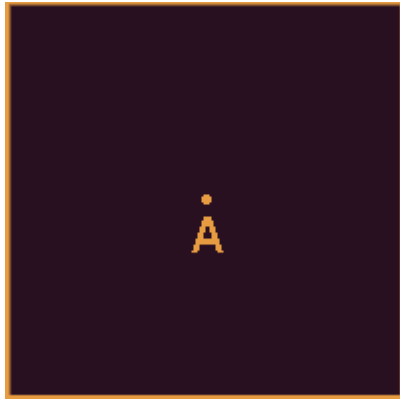
The Single Point is the first archetype of Sacred Geometry. It is directly related to unity consciousness or Oneness.... undivided God Mind.
The Single Point is the absolute root mental concept....the simplest idea possible. It is literally The First Dimension.... the omnipresent / omnipotent center.
Single Pointedness is the root of all holistic thinking.... THE ALL IS ONE.
Single Pointedness or Oneness, is that which is beond Duality
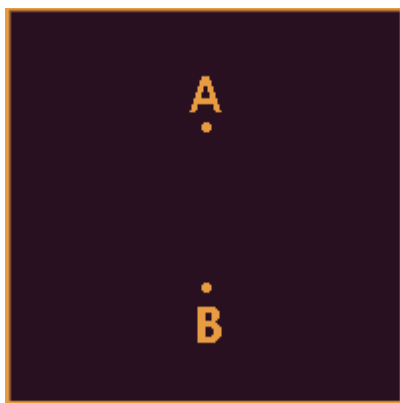

The two dimensional Universe begins in the division of The Single Point. This is The Great Mystery of Sacred Geometry: The Single Point magically divides and becomes The Two Points
The Universe is created by this division: unity becomes duality. This is the great miracle and mystery. Suddenly, Point A. is here and Point B. is there.
These two points, this first duality, marks the first architectural relationship of the Universe and creates the first abstract unit of measure, ie space. The Second Dimension literally begins at this conceptual level: The Two Points

The tremendous energies contained within this first two dimensional relationship of the Universe (The Two Points) manifests as a duality of motion: straight line motion (from Point A. to Point B.) and rotational motion (Point B. around Point A.). This dual movement is called The Radius/Arc.

This is the conceptual Big Bang. All the various energies of the universe trace to the play between the Radius and the Arc. The Radius/Arc is Yin and Yang... Light and Dark... Left and Right... Father and Mother... etc. All manifestations of duality trace to The Radius/Arc.
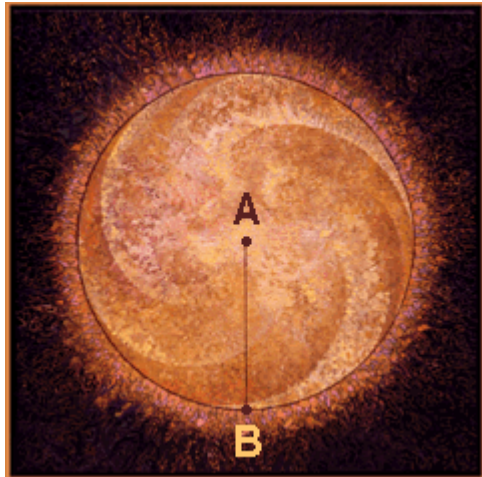
The timeless relationship forever held within The Radius/Arc (expressed scientifically as Pi - 3.1416....) is THE root mathematical formula and visually unfolds to become the first enclosed form of Sacred Geometry,... The Circle .

The Circle is unity... Oneness. It is the two dimentional manifestation of single pointedness.... undivided God Mind.

The Circle is... THE ALL. This is the essence of Mandala; The Circle holds ALL.

Definition: Pi: The ratio of the circumference of the circle to its diameter: a transcendental number having a value to eight places of 3.14159265. For practical purposes, the value of Pi is 3.1416.
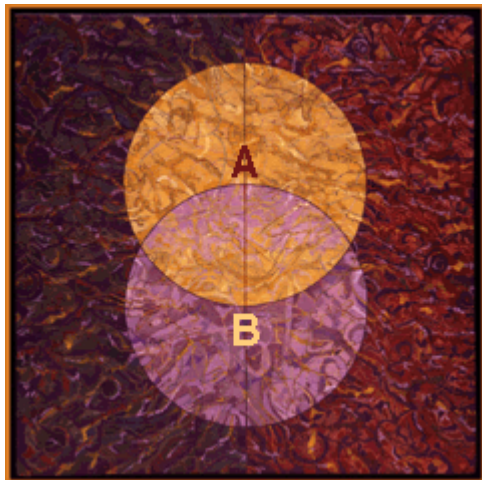
The first Icon (The Circle) is created by rotating point B. around point A. (the yellow circle).

But The Two Points, are perfect twins with equal potentials, and Point A. can also rotate around point B.... using the original radius.

This natural polality, this reversal of rolls, produces another circle (the violet one).

These two circles create the second enclosed form (overlaping) of Sacred Geometry entitled The Two Circles Of Common Radius.


These two overlapping circles with a common radius, create the third enclosed form of Sacred Geometry. The ancients called this archetype The Vesica Piscis. (black shape)

ALL the dimentional forms of this cosmos evolve from this football like shape... literaly ALL form. The Vesica Piscis is literally the womb of the universe.... the ever unfolding Mother of Sacred Geometry.
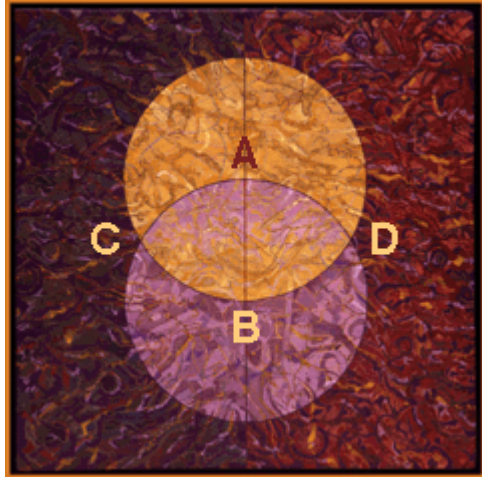
In addition to The Vesica Piscis, The Two Circles Of Common Radius create two new points at their intersections (C. & D.).

The play of Father/Mother points (A. and B.) create the first children of the universe: The twins (points C. & D.). And the magnification continues.

Rotating the first two points (A. & B.) around these new points (C. & D.) creates two additional circles and four additional Vesica Piscis. Now we have Four circles of common radius and Five Vesica Piscis (see Fig 8).

The two new circles also create four more points (E. F. G. & H.), and another enclosed form of Sacred Geometry we call, The Petal (the white shape at the center).

This is the germ of The Creation Pattern, as all of the necessarry forms are now in place: Parents (circles 1 & 2) and children (circles 3 & 4). This is the essence of family revealed in Sacred Geometry.... the family heart in the form of a petal.

And the magnification is about to explode!
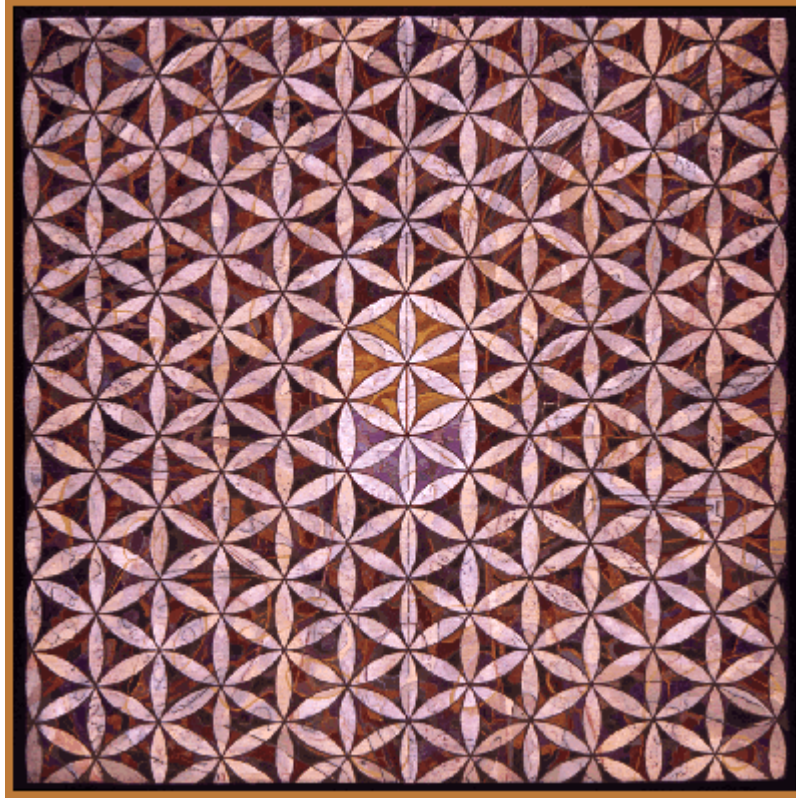
Striking more circles around the new points (E. F. G. & H.) compounds the forms and creates a never ending grid of Circles, Vesica Piscis, and Petals entitled Nature's First Pattern. This is an ever evolving two dimensional pattern which, at the conceptual level, circumnavigates the universe.

Nature's First Pattern is THE creation pattern. There are an infinite number of sub patterns and forms to be discovered within Nature's First Pattern, and the study of these unlimited possibilities IS the root two dimensional study of Sacred Geometry.

The Point

The point is found at the center of the sphere or the circle. All measurements must either begin with the point or pass through the point. It is the beginning and it is the end. In sacred geometry the center point is thought to be the place creation began.
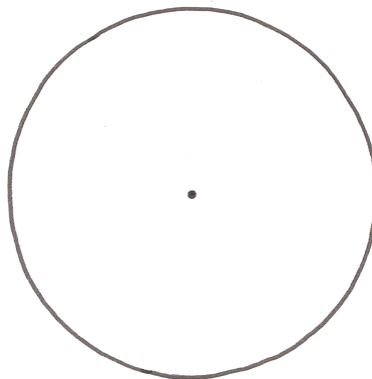
The Square Root of 2

The square root of 2 is an irrational number. When a square with sides that measure one unit is divided diagonally, the square root of 2 is the length of the diagonal. Like Pi, square root of 2 never ends. The total of the square root of 2 equals more than half of itself.

- [The Square Root of 2](#): A web page that shows the square root of 2 to 1 million digits completed at Nasa.

The Golden Ratio

The golden ratio, or phi, is the unique ratio in which the ratio of the larger portion is equal to the ratio of the smaller portion. The golden ratio is another irrational number. It is usually rounded to 1.618. It is also known as the golden mean, divine proportion, or golden section. The golden ratio has been used since ancient time in architecture of buildings.

The golden ratio, also known as the divine proportion, golden mean, or golden section, is a number often encountered when taking the ratios of distances in simple geometric figures such as the pentagram, decagon and dodecagon. It is denoted ϕ , or sometimes τ (which is a abbreviation of the Greek "tome," meaning "to cut").

The term "golden section" (*goldene Schnitt*) seems to first have been used by Martin Ohm in the 1835 2nd edition of his textbook *Die Reine Elementar-Mathematik* (Livio 2002, p. 6). The first known use of this term in English is in James Sulley's 1875 article on aesthetics in the 9th edition of the *Encyclopedia Britannica.* The symbol ("phi") was apparently first used by Mark Barr at the beginning of the 20th century in commemoration of the Greek sculptor Phidias (ca. 490-430 BC), who a number of art historians claim made extensive use of the golden ratio in his works (Livio 2002, pp. 5-6).

ϕ has surprising connections with continued fractions and the Euclidean algorithm for computing the greatest common divisor of two integers. It is also a so-called Pisot Number.
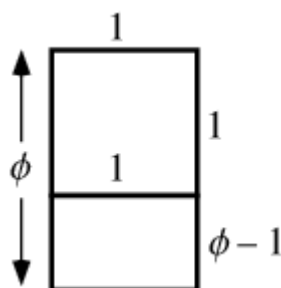
Given a rectangle having sides in the ratio , is defined such that partitioning the origin rectangle into a square and new rectangle results in a new rectangle having sides with a rati . Such a rectangle is called a golden rectangle, and successive points dividing a golden rectangle into squares lie on a logarithmic spiral. This figure is known as a whirling square. The legs of a golden triangle (an isosceles triangle with a vertex angle of) are in a golde ratio to its base and, in fact, this was the method used by Pythagoras to construct . The rat of the circumradius to the length of the side of a decagon is also ,

$$\frac{R}{s} = \tfrac{1}{2}\csc\left(\frac{\pi}{10}\right) = \tfrac{1}{2}\left(1+\sqrt{5}\right) = \phi.$$

Bisecting a (schematic) Gaullist cross also gives a golden ratio (Gardner 1961, p. 102).
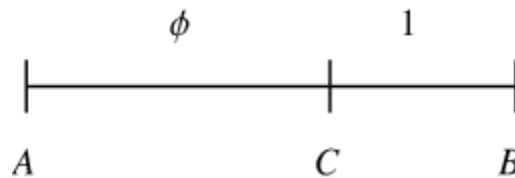
Euclid ca. 300 BC defined the "extreme and mean ratios" on a line segment as the lengths such that

$$\phi = \frac{AC}{CB} = \frac{AB}{AC}$$

(Livio 2002, pp. 3-4). Plugging in,

$$\frac{\phi + 1}{\phi} = \phi,$$

and clearing denominators gives

$$\phi^2 - \phi - 1 = 0.$$

(Incidentally, this means that is a algebraic number of degree 2.) So, using the quadratic equation and taking the positive sign (since the figure is defined so that $\phi > 1$),

$$\phi = \tfrac{1}{2}\left(1 + \sqrt{5}\right)$$

$$= 1.6180339887498948482045868343656381177203\ldots$$

(Sloane's A001622).

Exact trigonometric formulas for include

$$\phi = 2\cos\left(\frac{\pi}{5}\right)$$
$$= \frac{1}{2}\sec\left(\frac{2\pi}{5}\right)$$
$$= \frac{1}{2}\csc\left(\frac{\pi}{10}\right).$$

The golden ratio is given by the infinite series

$$\phi = \frac{13}{8} + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(2n+1)!}{(n+2)! \, n! \, 4^{2n+3}}$$

(B. Roselle). Another fascinating connection with the Fibonacci numbers is given by the infinite series

$$\phi = 1 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{F_n \, F_{n+1}}.$$

A representation in terms of a nested radical is

$$\phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$$

(Livio 2002, p. 83).

ϕ is the "most" irrational number because it has a continued fraction representation

$$\phi = [1, 1, 1, \ldots] = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$$

(Sloane's A000012; Williams 1979, p. 52; Steinhaus 1999, p. 45; Livio 2002, p. 84). This means that the convergents are given by the quadratic recurrence equation
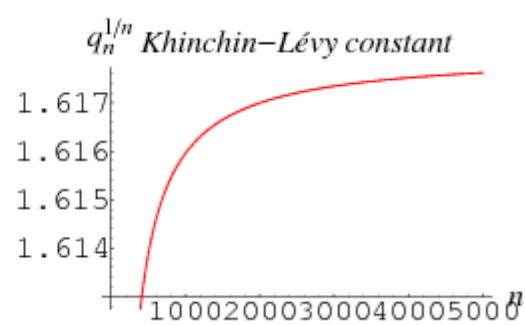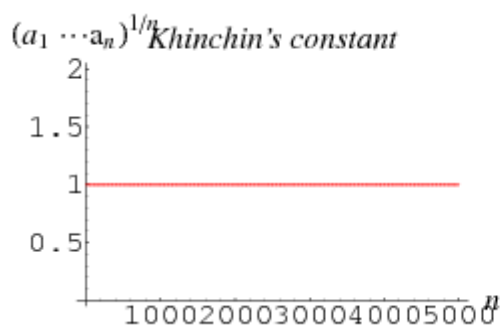
$$x_n = 1 + \frac{1}{x_{n-1}},$$

with $X_1 = 1$, which has solution

$$x_n = \frac{F_{n+1}}{F_n},$$

where F_2 is the n$^{\text{th}}$ Fibonacci number. As a result,

$$\phi = \lim_{n \to \infty} x_n = \lim_{n \to \infty} \frac{F_n}{F_{n-1}},$$

as first proved by Scottish mathematician Robert Simson in 1753 (Wells 1986, p. 62; Livio 2002, p. 101).

Let the continued fraction of be denoted and let the denominators of the convergents be denoted , , ..., . As can be seen from the plots above, the regularity in the continued fraction of means that is one of a set of numbers of measure 0 whose continued fraction sequences *do not* converge to the Khinchin constant or the Khinchin-Lévy constant.

The golden ratio has Engel expansion 1, 2, 5, 6, 13, 16, 16, 38, 48, 58, 104, ... (Sloane's A028259).

The golden ratio also satisfies the recurrence relation

$$\phi^n = \phi^{n-1} + \phi^{n-2}.$$

Taking $\eta = 1$ gives the special case

$$\phi = \phi^{-1} + 1.$$

Treating (◇) as a linear recurrence equation

$$\phi(n) = \phi(n-1) + \phi(n-2)$$

in $\phi(\eta) = \phi^{\eta}$, setting $\phi(0) = 1$ and $\phi(1) = \phi$, and solving gives

$$\phi(n) = \phi^n,$$

as expected. The powers of the golden ratio also satisfy

$$\phi^n = F_n \phi + F_{n-1},$$

where is a Fibonacci number (Wells 1986, p. 39).

The sine of certain complex numbers involving gives particularly simple answers, for Example

$$\sin(i \ln \phi) = \tfrac{1}{2} i$$
$$\sin(\tfrac{1}{2} \pi - i \ln \phi) = \tfrac{1}{2} \sqrt{5}$$

(D. Hoey, pers. comm.). A curious (although not particularly useful) approximation due to D Barron is given by

$$\phi \approx \tfrac{1}{2} K^{\gamma - 19/7} \pi^{2/7 + \gamma},$$

where is Catalan's constant and is the Euler-Mascheroni constant, which is good to two digits.
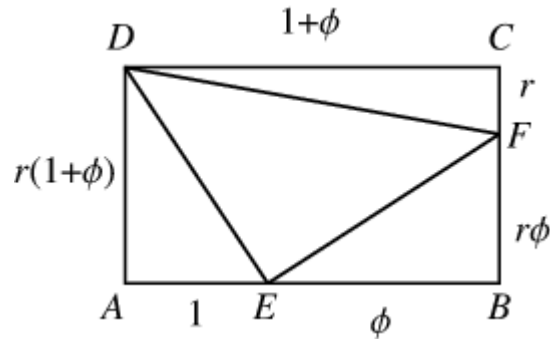
In the figure above, three triangles can be inscribed in the rectangle $ABCD$ of arbitrary aspect ratio $1 : r$ such that the three right triangles have equal areas by dividing AB and BC in the golden ratio. Then

$$K_{\triangle ADE} = \tfrac{1}{2} \cdot r (1 + \phi) \cdot 1 = \tfrac{1}{2} r \phi^2$$

$$K_{\triangle BEF} = \tfrac{1}{2} \cdot r \phi \cdot \phi = \tfrac{1}{2} r \phi^2$$

$$K_{\triangle CDF} = \tfrac{1}{2} (1 + \phi) \cdot r = \tfrac{1}{2} r \phi^2,$$

which are all equal.

The substitution map

$$0 \qquad \rightarrow \qquad 01$$
$$1 \qquad \rightarrow \qquad 0$$

Gives

$$0 \rightarrow 01 \rightarrow 010 \rightarrow 01001 \rightarrow \ldots,$$

giving rise to the sequence

01001010010010100101001001001 ...

(Sloane's A003849). Here, the zeros occur at positions 1, 3, 4, 6, 8, 9, 11, 12, ... (Sloane's A000201), and the ones occur at positions 2, 5, 7, 10, 13, 15, 18, ... (Sloane's A001950). These are complementary Beatty sequences generated by and . The sequence als has many connections with the Fibonacci numbers.

1	0.618034
2	0.236068
3	0.854102
4	0.472136
5	0.0901699
6	0.708204
7	0.326238
8	0.944272
9	0.562306
10	0.18034
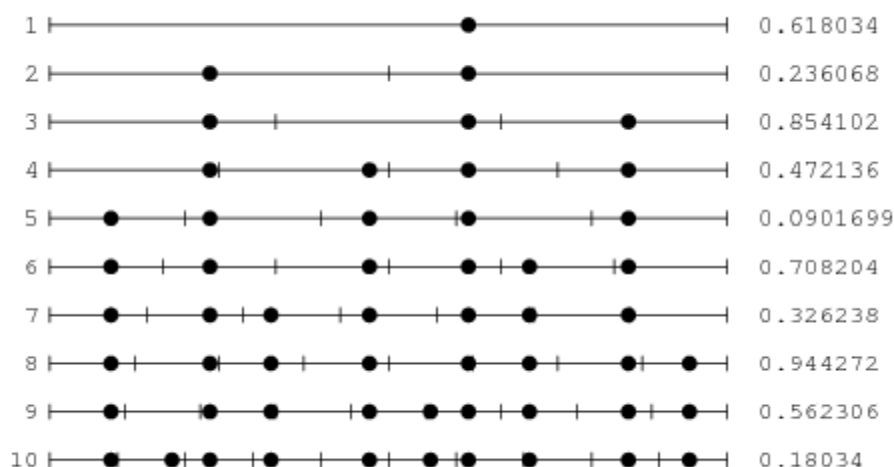
Steinhaus (1983, pp. 48-49) considers the distribution of the fractional parts of in the intervals bounded by 0, 1 /η, 2/η, ..., (η-1)/η, 1, and notes that they are much more uniformly distributed than would be expected due to chance (i.e., is close to an equidistributed sequence). In particular, the number of empty intervals for η=1, 2, ..., are mere 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 1, 1, 0, 2, 2, ... (Sloane's A036414). The values of for which *no* bins are left blank are then given by 1, 2, 3, 4, 5, 6, 8, 10, 13, 16, 21, 34, 55, 89, 144, ... (Sloane's A036415). Steinhaus (1983) remarks that the highly uniform distribution has its roots in the continued fraction for ϕ .

The sequence {frac (x^η)}, of power fractional parts, where is the fractional part, is equidistributed for *almost all* real numbers x>1, with the golden ratio being one exception.

Salem showed that the set of Pisot numbers is closed, with the smallest accumulation point of the set (Le Lionnais 1983).

SEE ALSO: Beraha Constants, Decagon, Equidistributed Sequence, Euclidean Algorithm, Five Disks Problem, Golden Angle, Golden Gnomon, Golden Ratio Conjugate, Golden Rectangle, Golden Triangle, Icosidodecahedron, Noble Number, Pentagon, Pentagram, Phi Number System, Phyllotaxis, Pisot Number, Power Fractional Parts, Ramanujan Continued Fractions, Rogers-Ramanujan Continued Fraction, Secant Method.

According to legend, the Greek Philosopher Pythagoras discovered the concept of harmony when he began his studies of proportion while listening to the different sounds given off when the blacksmith's hammers hit their anvils. The weights of the hammers and of the anvils all gave off different sounds. From here he moved to the study of stringed instruments and the different sounds they produced. He started with a single string and produced a monochord in the ratio of 1:1 called the Unison. By varying the string, he produced other chords: a ratio of 2:1 produced notes an octave apart. (Modern music theory calls a 5:4 ratio a "major third" and an 8:5 ratio a "major sixth".) In further studies of nature, he observed certain patterns and numbers reoccurring. Pythagoras believed that beauty was associated with the ratio of small integers.

Astonished by this discovery and awed by it, the Pythagoreans endeavored to keep this a secret; declaring that anybody that broached the secret would get the death penalty. With this discovery, the Pythagoreans saw the essence of the cosmos as numbers and numbers took on special meaning and significance.

The symbol of the Pythagorean brotherhood was the pentagram, in itself embodying several Golden Means.

The Greeks, who called it the Golden Section, based the entire design of the Parthenon on this proportion.

**The Greeks knew
it as the
<u>Golden Section</u>**

**and used it for beauty
and balance in the
design of architecture**

Phidias (500 BC - 432 BC), a Greek sculptor and mathematician, studied phi and applied it to the design of sculptures for the Parthenon.

Porch of Maidens, Acropolis, Athens

Euclid proved that the diagonals of the regular pentagon cut each other in "extreme and mean ratio", now more commonly known as the golden ratio. Here we represent the golden ratio by phi. F_n is the nth Fibonacci number.
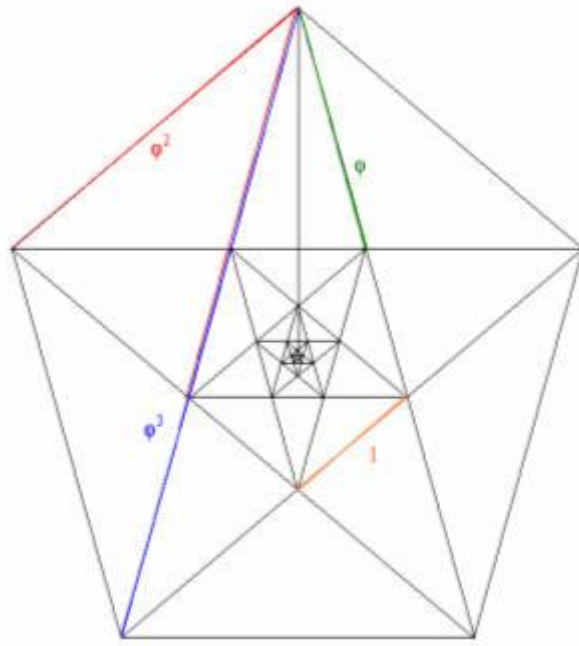
$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}$$

$$\varphi = \lim_{n \to \infty} \frac{F_{n+1}}{F_n}$$

$$\varphi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$$

$$\varphi^2 = \varphi + 1$$

$$\varphi = 1 + 1/\varphi$$

$$\varphi = \frac{1 + \sqrt{5}}{2}$$

The Square Root of 3 and the Vesica Piscis

The square root of 3 is a positive real number. When it is multiplied by itself it equals 3. The vesica picis is the name for the almond shaped area that is created when two circles of the same radius which intersect so that each circle lies within the circumference of the other. The geometric ratio of the almond space area is the square root of 3. It is considered to be the symbol for Jesus, part of the Ark of the Covenant along with other sacred meanings.

The **square root of 3** is the positive real number that, when multiplied by itself, gives the number 3.

The first sixty significant digits of its decimal expansion are:
 1.73205 08075 68877 29352 74463 41505 87236 69428 05253 81038 06280 5580...

The rounded value of 1.732 is correct to within 0.01% of the actual value.

The VESICA PISCIS
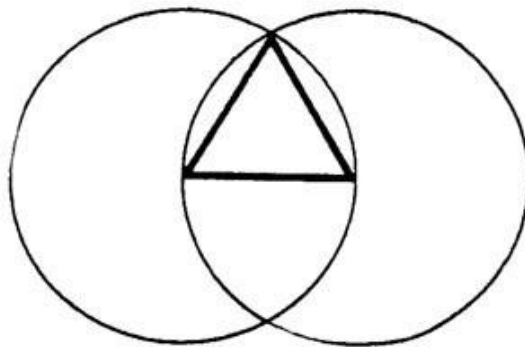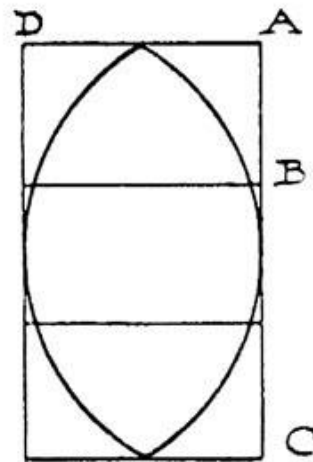
FIG 1   FIG. 2

The oval form of the VIII° symbol is of great significance and has been known for many centuries as the Vesica Piscis, or fish's bladder, probably to conceal its true meaning.

If we go back to the early stages of symbolism we find the circle used to represent eternity, and to man's conception, there are two eternities; that of the ages past, and that of the future. Two equal circles would symbolize these infinite periods of time and their intersection forms a cavity, which represents that attainment, which is the aim and object of every man's sojourn on this earth, and which in Christian symbolism, is called the birth and life of Christ in the soul.

The Vesica Piscis is an almost universal symbol for the Christ and this accounts for the use of the oval form on seals of ecclesiastical institutions.

The early Christians were anxious to include symbols of their religion in their cathedrals and churches, and this is the reason for the rise of Gothic architecture in which the windows were built in a pointed form to include this symbol of the Christ.

As the new covenant of the Avatar Jesus is said to have superseded that of Moses, so do we find the Christian degrees in Masonry beyond the craft or universal degrees, and in the same way Gothic architecture came after the square or Norman type.

The diagram shown above is the figure for the first proposition of the first book of Euclid, and all the subsequent propositions depend upon this fundamental one, which is to describe an equilateral triangle on a given line. When described in this way, the triangle forms a fitting symbol of the Christian Trinity.

If we enclose the Vesica Piscis in a rectangle, and divide that rectangle into three equal parts as shown in Fig. 2, we find that the ratio of the sides of one of the smaller triangles is equal to the ratio of the sides of the whole rectangle. In other words, the ratio of AB to AD is the same as the ratio of AD to AC, which ratio is that of 1 to the square root of 3. This is very important and significant fact and makes the symbol particularly referable to the Triune God.

This fact was evidently known to the builders of the Middle Ages for we find that the naves of many of the old cathedrals and churches were built with their sides in this same proportion. Rosslyn Chapel, in England is a famous example of this ratio.

Spirals

There are a number of different types of spirals. There are flat spirals, 3-D spirals, right-handed spirals, left-handed spirals, equi-angular spirals, geometric spirals, logarithmic spirals and rectangular spirals. The most well known spiral is that of the nautilus shell. All spirals have two things in common: expansion and growth. They are symbols of infinity.

Equiangular Spiral
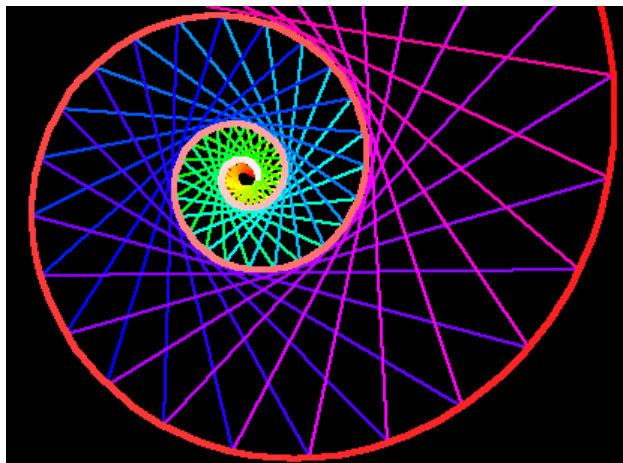
A equiangular spiral and its secants.

History

Want to learn differential equations? Our conceptual approach is your best bet. Visit [Differential Equations, Mechanics, and Computation](#)

The investigation of spirals began at least with the ancient Greeks. The famous Equiangular Spiral was discovered by [Rene Descartes](#), its properties of self-reproduction by [Jacob Bernoulli](#) (1654-1705) (aka James or Jacques) who requested that the curve be engraved upon his tomb with the phrase "Eadem mutata resurgo" ("I shall arise the same, though changed.") [Source: Robert C Yates (1952)]

The equiangular spiral was first considered in 1638 by Descartes, who started from the property s = a.r. [Evangelista Torricelli](#), who died in 1647, worked on it independently and used for a definition the fact that the radii are in geometric progression if the angles increase uniformly. From this he discovered the relation s = a.r; that is to say, he found the rectification of the curve. Jacob Bernoulli, some fifty years later, found all the "reproductive" properties of the curve; and these almost mystic properties of the "wonderful" spiral made him wish to have the curve incised on his tomb: Eadem mutata resurgo — "Though changed I rise unchanged". [source: E H Lockwood (1961)]

Description

Equiangular spiral describes a family of spirals of one parameter. It is defined as a curve that cuts all radial line at a constant angle.

It also called logarithmic spiral, Bernoulli spiral, and logistique.

Explanation:

1. Let there be a spiral (that is, any curve $r==f[\theta]$ where f is a [monotonic](#) inscreasing function)
2. From any point P on the spiral, draw a line toward the center of the spiral. (this line is called the radial line)
3. If the angle formed by the radial line and the tangent for any point P is constant, the curve is a equiangular spiral.
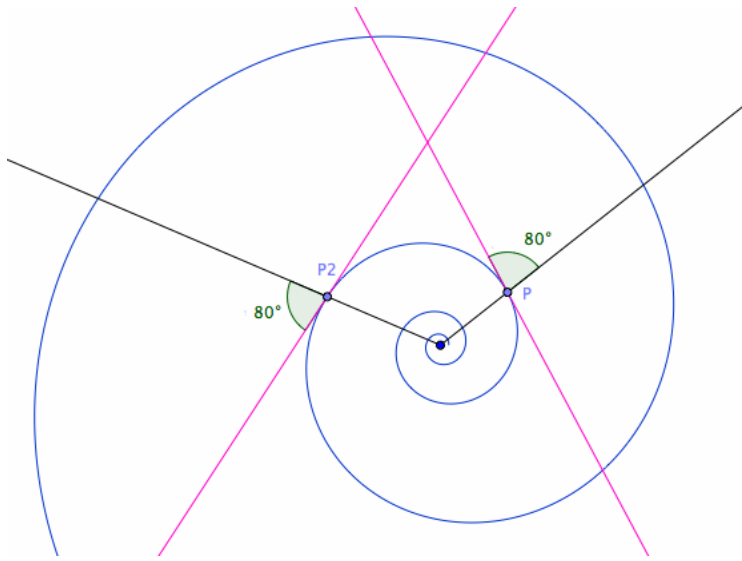
A example of equiangular spiral with angle 80°.

A special case of equiangular spiral is the circle, where the constant angle is 90°.
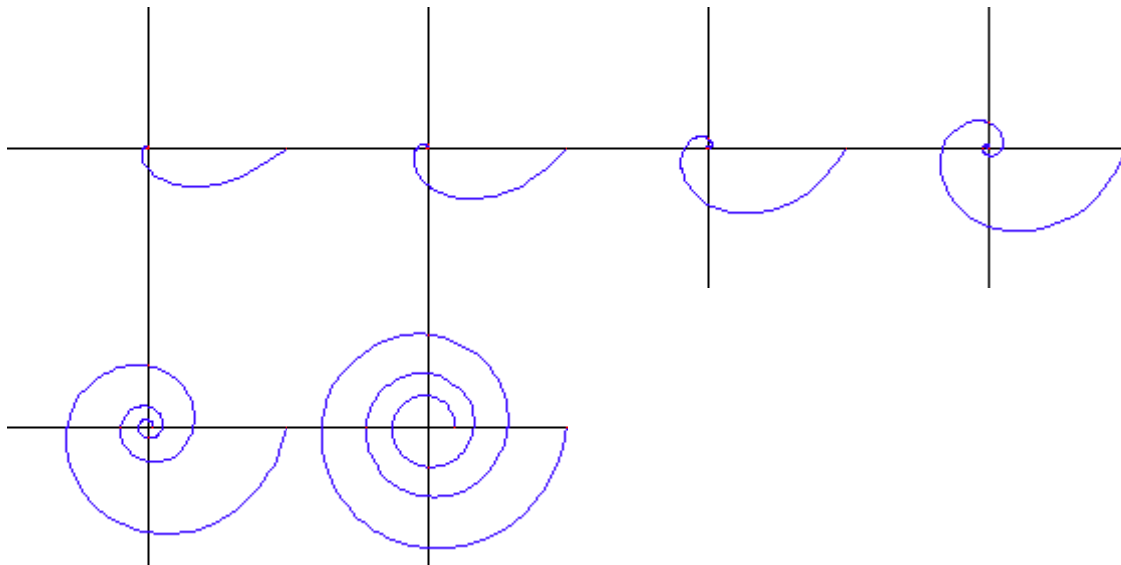
Equiangular spirals with 40°, 50°, 60°, 70°, 80° and 85°. (left to right) Equiangular Spiral

Formulas

Let α be the constant angle.
Polar: r == E^(θ * Cot[α]) equiangular_spiral.gcf
Parametric: E^(t * Cot[α]) {Cos[t],Sin[t]}
Cartesian: x^2 + y^2 == E^(ArcTan[y/x] Cot[α])

Properties

Point Construction and Geometric Sequence

Length of segments of any radial ray cut by the curve is a geometric sequence, with a multiplier of E^(2 π Cot[α]).

Lengths of segments of the curve, cut by equally spaced radial rays, is a geometric sequence.
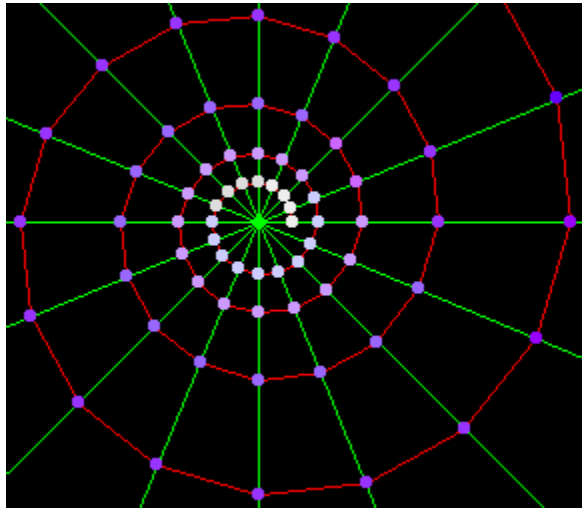
The curve cut by radial rays. The length of any green ray's segments is geometric sequence. The lengths of red segments is also a geometric sequence. In the figure, the dots are points on a 85° equiangular spiral.

Catacaustic

Catacaustic of a equiangular spiral with light source at center is a equal spiral.

Proof: Let O be the center of the curve. Let α be the curve's constant angle. Let Q be the reflection of O through the tangent normal of a point P on the curve. Consider Triangle[O,P,Q]. For any point P, Length[Segment[O,P]]==Length[Segment[P,Q]] and Angle[O,P,Q] is constant. (Angle[O,P,Q] is constant because the curve's constant angle definition.) Therefore, by argument of similar triangle, then for any point P, Length[Segment[O,Q]]==Length[Segment[O,P]]*s for some constant s. Since scaling and rotation around its center does not change the curve, thus the locus of Q is a equiangular spiral with constant angle α, and Angle[O,Q,P] == α. Line[P,Q] is the tangent at Q.
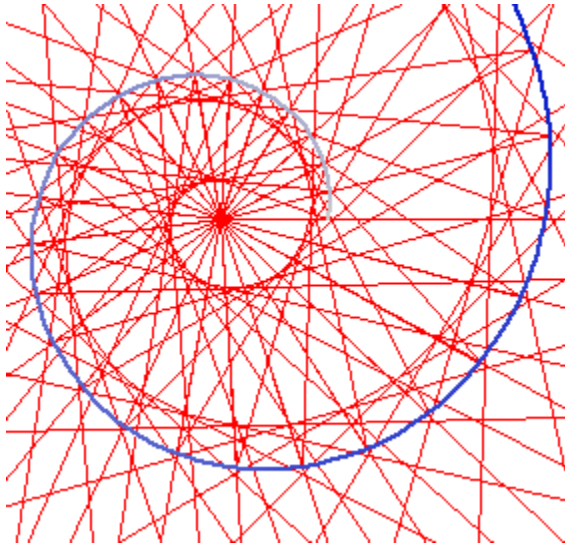
Equiangular Spiral Caustic

Curvature

The evolute of a equiangular spiral is the same spiral rotated.

The involute of a equiangular spiral is the same spiral rotated.
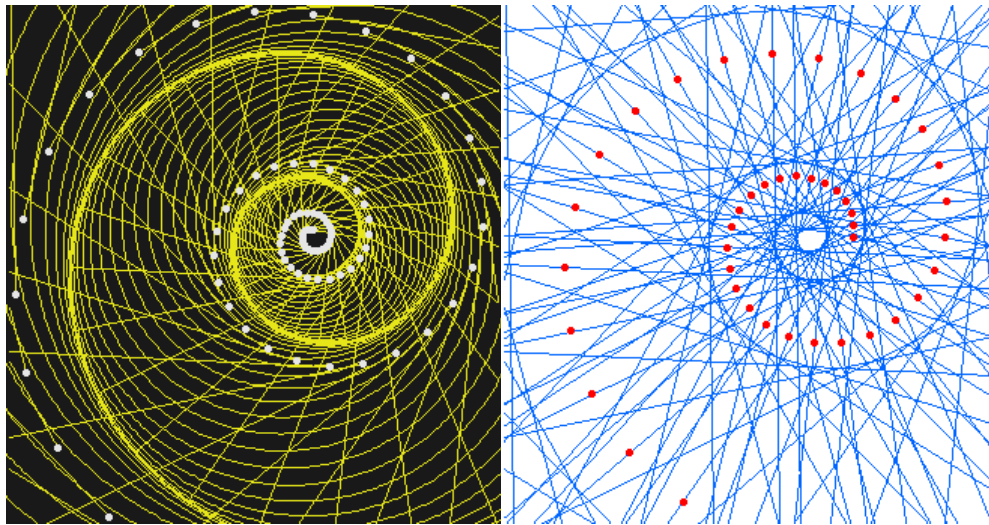
Left: Tangent circles of a 80° equiangular spiral. The white dots are the centers of tangent circles, the lines are the radiuses. Right: Lines are the tangent normals, forming the evolute curve by envelope. Equiangular Spiral Evolute

Radial

The radial of a equiangular spiral is itself scaled. The figure on the left shows a 70° equiangular spiral and its radial. The figure on the right shows its involute, which is another equiangular spiral.
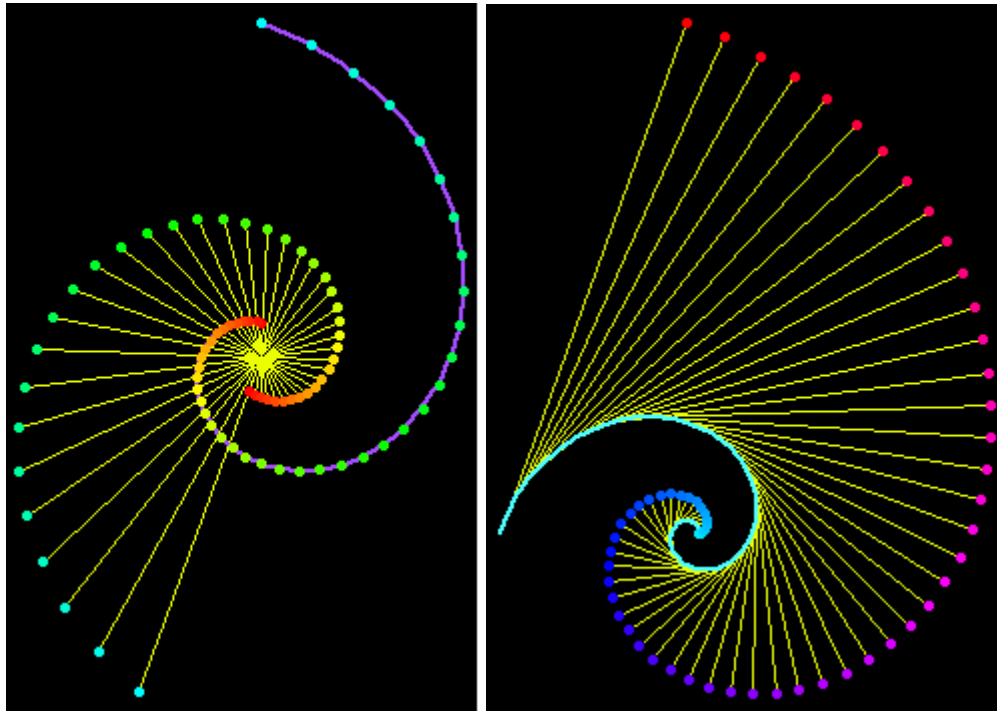
Inversion

The inversion of a equiangular spiral with respect to its center is a equal spiral.
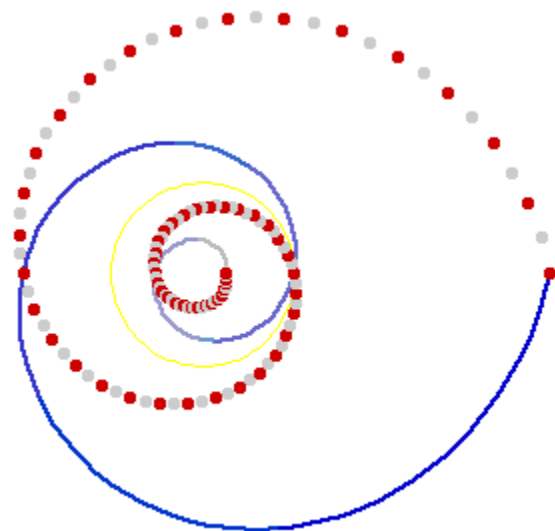
Pedal

The [pedal](#) of a equiangular spiral with respect to its center is a equal spiral.
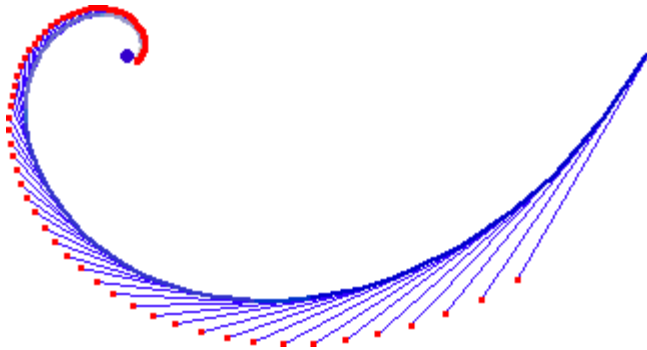
Pedal of a equiangular spiral. The lines from center to the red dots is perpendicular to the tangents (blue lines). The blue curve is a 60° equiangular spiral. The red dots forms its pedal.

Pursuit Curve

Persuit curves are the trace of a object chasing another. Suppose there are n bugs each at a corner of a n sided regular polygon. Each bug crawls towards its next neighbor with uniform speed. The trace of these bugs are equiangular spirals of (n-2)/n * π/2 radians (half the angle of the polygon's corner).
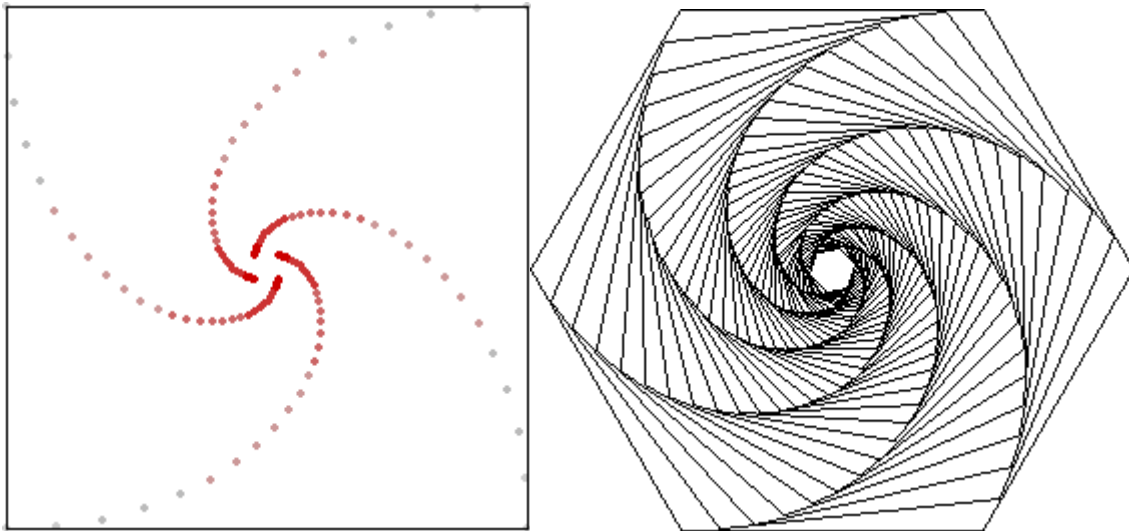
Left: shows the trace of four bugs, resulting four equiangular spirals of 45°. Above right: six objects forming a chasing chain. Each line is the direction of movement and is tangent to the equiangular spirals so formed.

Spiral in nature

Spiral is the basis for many natural growths.

Seashells have the geometry of equiangular spiral. See: Mathematics of Seashell Shapes.

A cauliflower (Romanesco broccoli) exhibiting equiangular spiral and fractal geometry. (Photo by Dror Bar-Natan. Source)

Spirals

- Belousov's Brew. A recipe for making spiraling patterns in chemical reactions.
- Equiangular spiral. Properties of Bernoulli's logarithmic 'spiralis mirabilis'.
- Fermat's spiral and the line between Yin and Yang. Taras Banakh, Oleg Verbitsky, and Yaroslav Vorobets argue that the ideal shape of the dividing line in a Yin-Yang symbol is formed, not from two semicircles, but from Fermat's spiral.

- [Fourier series of a gastropod](). L. Zucca uses Fourier analysis to square the circle and to make an odd spiral-like shape.
- [The golden bowls and the logarithmic spiral]().
- [Golden spiral flash animation](), Christian Stadler.
- [Graphite with growth spirals on the basal pinacoids](). Pretty pictures of spirals in crystals. (A pinacoid, it turns out, is a plane parallel to two crystallographic axes.)
- [Helical Gallery](). Spirals in the work of M. C. Escher and in X-ray observations of the sun's corona.
- [Mathematical imagery by Jos Leys](). Knots, Escher tilings, spirals, fractals, circle inversions, hyperbolic tilings, Penrose tilings, and more.
- [Log-spiral tiling](), and other [radial and spiral tilings](), S. Dutch.
- [Looking at sunflowers](). In this abstract of an undergraduate research paper, Surat Intasang investigates the spiral patterns formed by sunflower seeds, and discovers that often four sets of spirals can be discerned, rather than the two sets one normally notices.
- [Modeling mollusc shells with logarithmic spirals](), O. Hammer, Norsk Net. Tech. Also includes a list of logarithmic spiral links.
- [Pi curve](). Kevin Trinder squares the circle using its involute spiral. See also his [quadrature based on the 3-4-5 triangle]().
- [Pictures of various spirals](), Eric Weeks.
- [Polyform spirals]().
- [Ram's Horn]() cardboard model of an interesting 3d spiral shape bounded by a helicoid and two nested cones.
- [Research: spirals](), Mícheál Mac an Airchinnigh. Presumably this connects to his thesis that "there is a geometry of curves which is computationally equivalent to a Turing Machine".
- [Seashell spirals](). Xah Lee examines the shapes of various real seashells, and offers prize money for formulas duplicating them.
- [Soddy Spiral](). R. W. Gosper calculates the positions of a sequence of circles, each tangent to the three previous ones.
- [Spidron](), a triangulated double spiral shape tiles the plane and various other surfaces. With photos of related paperfolding experiments.
- [Spira Mirabilis]() logarithmic spiral applet by A. Bogomily.
- [Spiral generator](), web form for creating bitmap images of colored logarithmic spirals.

- [Spiral in a liquid crystal film]().
- [Spiral minaret of Samara]().

- [A spiral of squares with Fibonacci-number sizes](), closely related to the golden spiral, Keith Burnett. See also his [hand-painted Taramundi spiral]().
- [Spiral tea cozy](), Kathleen Sharp.
- [Spiral tilings](). These similarity tilings are formed by applying the exponential function to a lattice in the complex number plane.

- [Spiral tower](). Photo of a building in Iraq, part of a web essay on the geometry of cyberspace.
- [Spiral triangles](), Eric Weeks.
- [Spiraling Sphere Models](). Bo Atkinson studies the geometry of a solid of revolution of an Archimedean spiral.
- [Spirals](). Mike Callahan and Larry Shook use a spreadsheet to investigate the spirals formed by repeatedly nesting squares within larger squares.
- [Spirals]() and other [2d curves](), Jan Wassenaar.
- [Spring into action](). Dynamic origami. Ben Trumbore, based on a model by Jeff Beynon from Tomoko Fuse's book *Spirals*.
- These two pictures by Richard Phillips are from the now-defunct *maths with photographs* website. The chimney is (Phillips thinks) somewhere in North Nottinghamshire, England. [A similar collection of Phillips' mathematical photos is now available on CD-ROM.]()

- [Three]() [spiral]() [tattoos]() from the Discover Magazine [Science Tattoo Emporium]().
- [The uniform net (10,3)-a](). An interesting crystal structure formed by packing square and octagonal helices.

- , T. Reluga. This term paper for a course on Greek science includes sections on the three classical problems, the Pythagorean theorem, the golden ratio, and the Archimedean spiral.

Toroids

A toroid is a circular shaped object, such as an o-ring. It is formed through repeated circular rotations. Each circle meets in the center of the toroid. A popular childhood toy, a spirograph, can be used to create one.
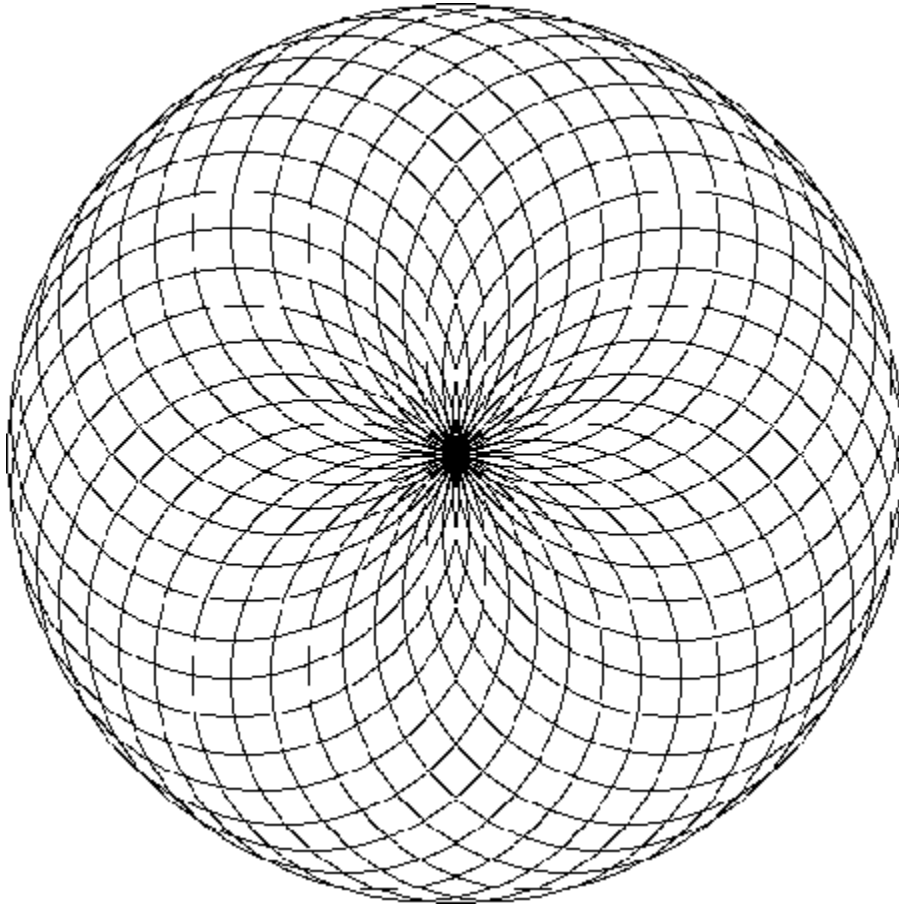
Rotating a circle about a line tangent to it creates a torus, which is similar to a donut shape where the center exactly touches all the "rotated circles." The surface of the torus can be covered with 7 distinct areas, all of which touch each other; an example of the classic "map problem" where one tries to find a map where the least number of unique colors are needed. In this 3-dimensional case, 7 colors are needed, meaning that the torus has a high degree of "communication" across its surface. The image shown is a

Dimensionality

We see things in either 2 or 3 dimensions. But what about a 4th dimension? Physics debates whether we exist within 3 or 4 dimension. Sacred geometry takes all 4 dimensions into consideration.

The progression from point (0-dimensional) to line (1-dimensional) to plane (2-dimensional) to space (3-dimensional) and beyond leads us to the question - if mapping from higher order dimensions to lower ones loses vital information (as we can readily observe with optical illusions resulting from third to second dimensional mapping), does our "fixation" with a 3-dimensional space introduce crucial distortions in our view of reality that a higher-dimensional perspective would not lead us to?

Fractals and Recursive Geometries

Fractals are a relatively new form of mathematics, beginning only in the 17th century. A good example of a fractal form is a fern. Each leaf on a fern is made up of smaller leaves that have the same shape of the larger whole. In recursive geometry the formula making up a form can be used repeatedly.

Most physical systems of nature and many human artifacts are not regular geometric shapes of the standard geometry derived from Euclid. Fractal geometry offers almost unlimited ways of describing, measuring and predicting these natural phenomena. But is it possible to define the whole world using mathematical equations?

This article describes how the four most famous fractals were created and explains the most important fractal properties, which make fractals useful for different domain of science.

Fractals' properties

Two of the most important properties of fractals are self-similarity and non-integer dimension.

What does self-similarity mean? If you look carefully at a fern leaf, you will notice that every little leaf □ part of the bigger one □ has the same shape as the whole fern leaf. You can say that the fern leaf is self-similar. The same is with fractals: you can magnify them many times and after every step you will see the same shape, which is characteristic of that particular fractal.

The non-integer dimension is more difficult to explain. Classical geometry deals with objects of integer dimensions: zero dimensional points, one dimensional lines and curves, two dimensional plane figures such as squares and circles, and three dimensional solids such as cubes and spheres. However, many natural phenomena are better described using a dimension between two whole numbers. So while a straight line has a dimension of one, a fractal curve will have a dimension between one and two, depending on how much space it takes up as it twists and curves. The more the flat fractal fills a plane, the closer it approaches two dimensions. Likewise, a "hilly fractal scene" will reach a dimension somewhere between two and three. So a fractal landscape

made up of a large hill covered with tiny mounds would be close to the second dimension, while a rough surface composed of many medium-sized hills would be close to the third dimension.

There are a lot of different types of fractals. In this paper I will present two of the most popular types: complex number fractals and Iterated Function System (IFS) fractals.

Complex number fractals

Before describing this type of fractal, I decided to explain briefly the theory of complex numbers.

A complex number consists of a real number added to an imaginary number. It is common to refer to a complex number as a "point" on the complex plane. If the complex number is $Z = (a + b \cdot i)$, the coordinates of the point are a (horizontal □ real axis) and b (vertical □ imaginary axis).

The unit of imaginary numbers: $i = \sqrt{(-1)}$.

Two leading researchers in the field of complex number fractals are Gaston Maurice Julia and Benoit Mandelbrot.

Gaston Maurice Julia was born at the end of 19th century in Algeria. He spent his life studying the iteration of polynomials and rational functions. Around the 1920s, after publishing his paper on the iteration of a rational function, Julia became famous. However, after his death, he was forgotten.

In the 1970s, the work of Gaston Maurice Julia was revived and popularized by the Polish-born Benoit Mandelbrot. Inspired by Julia's work, and with the aid of computer graphics, IBM employee Mandelbrot was able to show the first pictures of the most beautiful fractals known today.

Mandelbrot set

The Mandelbrot set is the set of points on a complex plain. To build the Mandelbrot set, we have to use an algorithm based on the recursive formula:

$$Z_n = Z_{n-1}^2 + C,$$

separating the points of the complex plane into two categories:

- points inside the Mandelbrot set,
- points outside the Mandelbrot set.

The image below shows a portion of the complex plane. The points of the Mandelbrot set have been colored black.
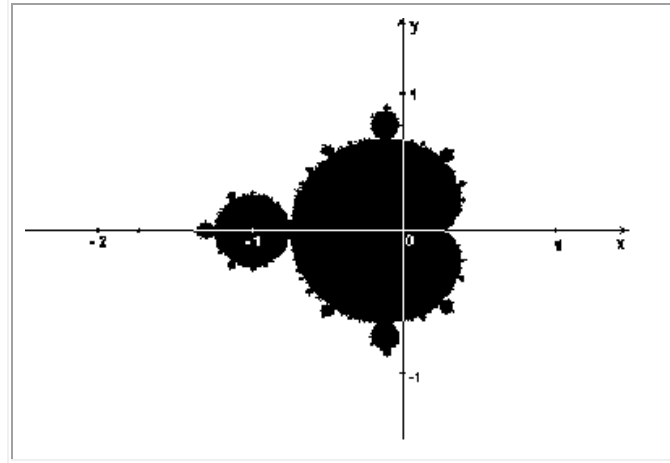
It is also possible to assign a color to the points outside the Mandelbrot set. Their colors depend on how many iterations have been required to determine that they are outside the Mandelbrot set.
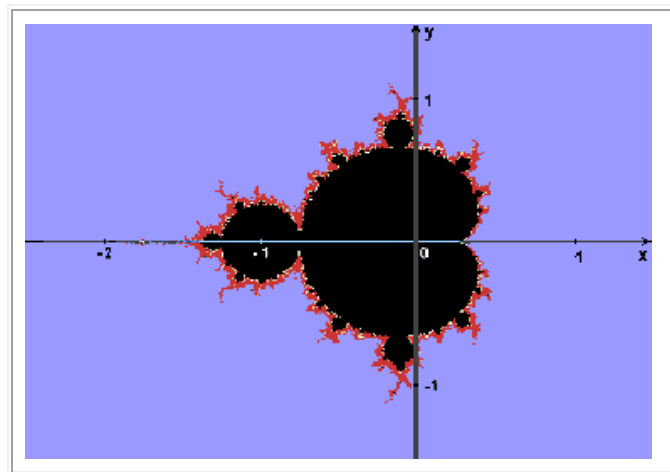
How is the Mandelbrot set created?

To create the Mandelbrot set we have to pick a point (C) on the complex plane. The complex number corresponding with this point has the form: $C = a + b \cdot i$

After calculating the value of previous expression:

$$Z_1 = Z_0^2 + C$$

using zero as the value of Z_0, we obtain C as the result. The next step consists of assigning the result to Z_1 and repeating the calculation: now the result is the complex number $C_2 + C$. Then we have to assign the value to Z_2 and repeat the process again and again.

This process can be represented as the "migration" of the initial point C across the plane. What happens to the point when we repeatedly iterate the function? Will it remain near to the origin or will it go away from it, increasing its distance from the origin without limit? In the first case, we say that C belongs to the Mandelbrot set (it is one of the black points in the image); otherwise, we say that it goes to infinity and we assign a color to C depending on the speed at which the point "escapes" from the origin.

We can take a look at the algorithm from a different point of view. Let us imagine that all the points on the plane are attracted by both: infinity and the Mandelbrot set. That makes it easy to understand why:

- points far from the Mandelbrot set rapidly move towards infinity,
- points close to the Mandelbrot set slowly escape to infinity,
- points inside the Mandelbrot set never escape to infinity.

Julia sets

Julia sets are strictly connected with the Mandelbrot set. The iterative function that is used to produce them is the same as for the Mandelbrot set. The only difference is the way this formula is used. In order to draw a picture of the Mandelbrot set, we iterate the formula for each point C of the complex plane, always starting with $Z_0 = 0$. If we want to make a picture of a Julia set, C must be constant during the whole generation process, while the value of Z_0 varies. The value of C determines the shape of the Julia set; in other words, each point of the complex plane is associated with a particular Julia set.

How is a Julia set created?

We have to pick a point C) on the complex plane. The following algorithm determines whether or not a point on complex plane Z) belongs to the Julia set associated with C, and determines the color that should be assigned to it. To see if Z belongs to the set, we have to iterate the function $Z_1 = Z_0^2 + C$ using $Z_0 = Z$. What happens to the initial point Z when the formula is iterated? Will it remain near to the origin or will it go away from it, increasing its distance from the origin without limit? In the first case, it belongs to the Julia set; otherwise it goes to infinity and we assign a color to Z depending on the speed the point "escapes" from the origin. To produce an image of the whole Julia set associated with C, we must repeat this process for all the points Z whose coordinates are included in this range:

$$-2 < a < 2; \; -1,5 < y < 1,5$$

The most important relationship between Julia sets and Mandelbrot set is that while the Mandelbrot set is connected (it is a single piece), a Julia set is connected only if it is associated with a point inside the Mandelbrot set. For example: the Julia set associated with C_1 is connected; the Julia set associated with C_1 is not connected (see picture below).
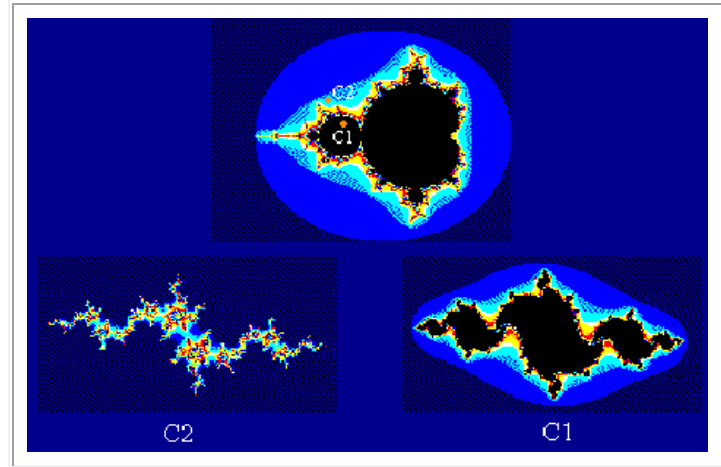
Iterated Function System Fractals

Iterated Function System (IFS) fractals are created on the basis of simple plane transformations: scaling, dislocation and the plane axes rotation. Creating an IFS fractal consists of following steps:

1. defining a set of plane transformations,
2. drawing an initial pattern on the plane (any pattern),
3. transforming the initial pattern using the transformations defined in first step,
4. transforming the new picture (combination of initial and transformed patterns) using the same set of transformations,
5. repeating the fourth step as many times as possible (in theory, this procedure can be repeated an infinite number of times).

The most famous ISF fractals are the Sierpinski Triangle and the Koch Snowflake.

Sierpinski Triangle

This is the fractal we can get by taking the midpoints of each side of an equilateral triangle and connecting them. The iterations should be repeated an infinite number of times. The pictures below present four initial steps of the construction of the Sierpinski Triangle:
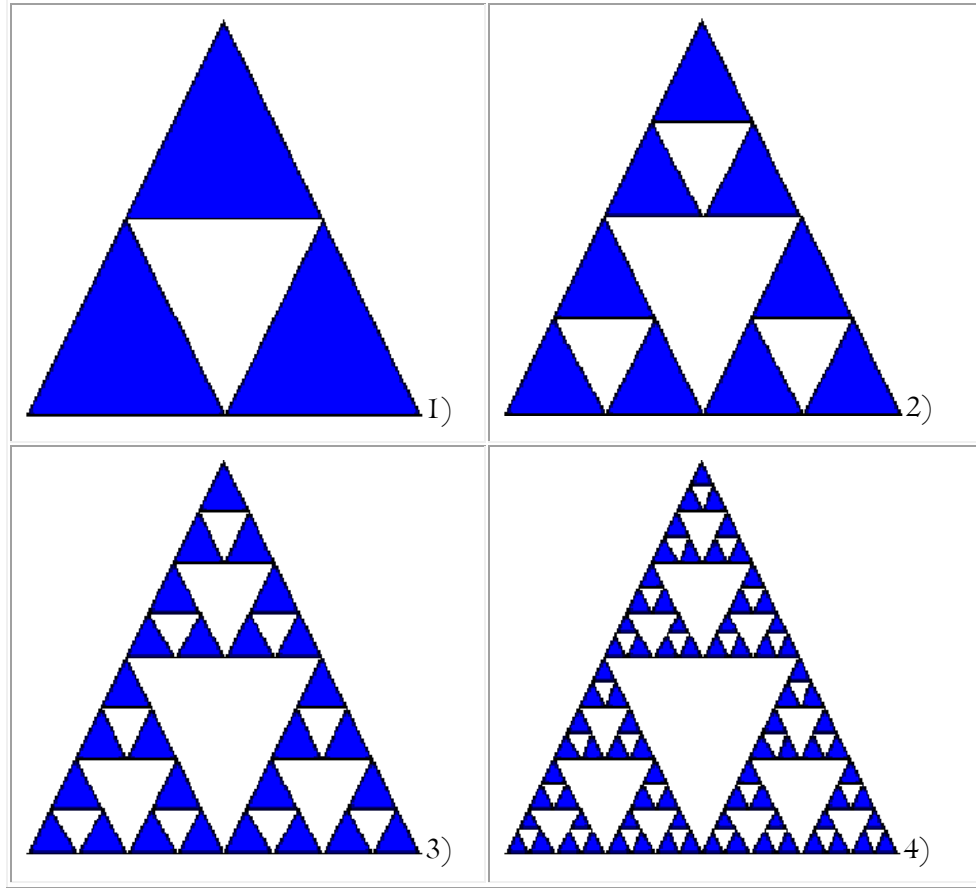
Using this fractal as an example, we can prove that the fractal dimension is not an integer.

First of all we have to find out how the "size" of an object behaves when its linear dimension increases. In one dimension we can consider a line segment. If the linear dimension of the line segment is doubled, then the length (characteristic size) of the line has doubled also. In two dimensions, if the linear dimensions of a square for example is doubled then the characteristic size, the area, increases by a factor of *4*. In three dimensions, if the linear dimension of a box is doubled then the volume increases by a factor of *8*.

This relationship between dimension *D*, linear scaling *L* and the result of size increasing *S* can be generalized and written as:

$$S = L \cdot D$$

Rearranging of this formula gives an expression for dimension depending on how the size changes as a function of linear scaling:

$$D = \frac{\log(S)}{\log(L)}$$

In the examples above the value of D is an integer — *1, 2*, or *3* — depending on the dimension of the geometry. This relationship holds for all Euclidean shapes. How about fractals?

Looking at the picture of the first step in building the Sierpinski Triangle, we can notice that if the linear dimension of the basis triangle (L) is doubled, then the area of whole fractal (blue triangles) increases by a factor of three (S).

Using the pattern given above, we can calculate a dimension for the Sierpinski Triangle:

$$D = \frac{\log(3)}{\log(2)} = 1,585$$

The result of this calculation proves the non-integer fractal dimension.

Koch Snowflake

To construct the Koch Snowflake, we have to begin with an equilateral triangle with sides of length, for example, *1*. In the middle of each side, we will add a new triangle one-third the size; and repeat this process for an infinite number of iterations. The length of the boundary is $3 \cdot \frac{4}{3} \cdot \frac{4}{3} \cdot \frac{4}{3} \cdot$ ...-infinity. However, the area remains less than the area of a circle drawn around the original triangle. That means that an infinitely long line surrounds a finite area. The end construction of a Koch Snowflake resembles the coastline of a shore.

Four steps of Koch Snowflake construction:
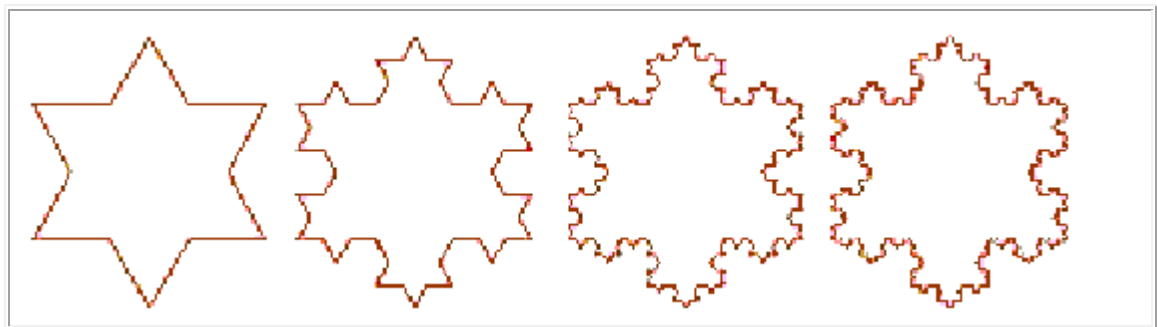
Another IFS fractals:

Fern leaf
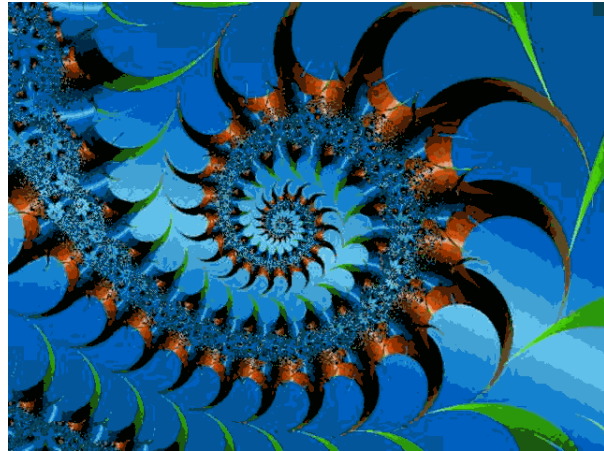
Spiral

Fractals applications

Fractal geometry has permeated many area of science, such as astrophysics, biological sciences, and has become one of the most important techniques in computer graphics.

Fractals in astrophysics

Nobody really knows how many stars actually glitter in our skies, but have you ever wondered how they were formed and ultimately found their home in the Universe? Astrophysicists believe that the key to this problem is the fractal nature of interstellar gas. Fractal distributions are hierarchical, like smoke trails or billowy clouds in the sky. Turbulence shapes both the clouds in the sky and the clouds in space, giving them an irregular but repetitive pattern that would be impossible to describe without the help of fractal geometry.

Fractals in the Biological Sciences

Biologists have traditionally modeled nature using Euclidean representations of natural objects or series. They represented heartbeats as sine waves, conifer trees as cones, animal habitats as simple areas, and cell membranes as curves or simple surfaces. However, scientists have come to recognize that many natural constructs are better characterized using fractal geometry. Biological systems and processes are typically characterized by many levels of substructure, with the same general pattern repeated in an ever-decreasing cascade.

Scientists discovered that the basic architecture of a chromosome is tree-like; every chromosome consists of many 'mini-chromosomes', and therefore can be treated as fractal. For a human chromosome, for example, a fractal dimension D equals *2,34* (between the plane and the space dimension).

Self-similarity has been found also in DNA sequences. In the opinion of some biologists fractal properties of DNA can be used to resolve evolutionary relationships in animals.

Perhaps in the future biologists will use the fractal geometry to create comprehensive models of the patterns and processes observed in nature.

Fractals in computer graphics

The biggest use of fractals in everyday live is in computer science. Many image compression schemes use fractal algorithms to compress computer graphics files to less than a quarter of their original size.

Computer graphic artists use many fractal forms to create textured landscapes and other intricate models.

It is possible to create all sorts of realistic "fractal forgeries" images of natural scenes, such as lunar landscapes, mountain ranges and coastlines. We can see them in many special effects in Hollywood movies and also in television advertisements. The "Genesis effect" in the film "Star Trek II - The Wrath of Khan" was created using fractal landscape algorithms, and in "Return of the Jedi" fractals were used to create the geography of a moon, and to draw the outline of the dreaded "Death Star". But fractal signals can also be used to model natural objects, allowing us to define mathematically our environment with a higher accuracy than ever before.

The Geometry of Fractal Shapes

- To explain the process by which **fractals** such as the Koch snowflake and the Sierpinski Gasket are constructed.
- To recognize self-similarity (or symmetry of scale) and its relevance.
- To describe how random processes can create **fractals** such as the Sierpinski Gasket.
- To explain the process by which the Mandelbrot set is constructed.

The Koch Snowflake (Recursive Construction)

- **Start.** Start with a solid equilateral triangle (a). The size of the triangle is irrelevant, so for simplicity we will say that the sides of the triangle are of length 1.

- **Step 1.** To the middle third of each of the sides of the original triangle add an equilateral triangle with sides of length 1/3, as shown in (b). The result is the 12-sided "star of David" shown in (c).

- **Step 2.** To the middle third of each of the 12 sides of the star in Step 1 add an equilateral triangle with sides of length one-third the length of that side.

- **Step 2 (cont).** The result is a "snowflake" with $12 \times 4 = 48$ sides, each of length $(1/3)^2 = 1/9$, as shown in (a). (Each of the sides "crinkles" into four new sides; each new side has length 1/3 the previous side.

- **Step 3.** Apply *Procedure KS* to the "snowflake" in Step 2. This gives the more elaborate "snowflake" shown in (b). Without counting we can figure out that this snowflake has $48 \times 4 = 192$ sides, each of length $(1/3)^3 = 1/27$

- **Step 4.** Apply *Procedure KS* to the "snowflake" in Step 3. This gives the "snowflake" shown in (c). (You definitely don't want to do this by hand– there are 192 tiny little equilateral triangles that are being added!)

- **Step 5, 6 etc.** Apply *Procedure KS* to the "snowflake" obtained in the previous step.

At each step of this process we create a new "snowflake", but after a while it's hard to tell that there is any change. For all practical purposes we are seeing the ultimate destination of this trip: the **Koch snowflake** itself as shown by the figure on the right.

The Koch snowflake is a fairly complicated shape, but we can define it in two lines using a form of shorthand we will call a replacement rule– a rule that specifies how to substitute one piece for another.

- **Start**: Start with a solid equilateral triangle $\triangle$.
- **Replacement Rule**: Whenever you see a boundary line segment, apply *Procedure KS* to it.

If we only consider the boundary of the Koch snowflake and forget about the interior, we get an infinitely jagged curve known as the Koch curve (or sometimes called the *snowflake curve*) shown in (a).

Clearly (a) is just a rough rendering of the Koch curve, so our natural curiosity pushes us to take a closer look. We'll just randomly pick a small section of the Koch curve and magnify it (b).

The surprise (or not!) is that we see nothing new– the small detail looks just like the rough detail. Figure (c) shows a detail of the Koch curve after magnifying it by a factor of almost 100.

To compute the boundary of the Koch snowflake, let's look at the boundary of the figures obtained in steps 1 and 2 of the construction in the above figure. At each step we replace a side by four sides that are 1/3 as long.

Thus, at any given step the perimeter is 4/3 times the perimeter at the preceding step. This implies that the perimeters keep growing with each step, and growing very fast indeed. Therefore,

- **The Koch snowflake has infinite perimeter.**

To compute the exact area of the Koch snowflake is considerably more difficult, but, as we see from the above figure, the Koch snowflake fits inside the circle that circumscribes the original equilateral triangle. Therefore,

- **The area of the Koch snowflake is 1.6 times the area of the starting equilateral triangle.**

The Sierpinski Gasket

- **Plurality method**

Election of 1st place votes

- **Plurality candidate**

The Candidate with the most 1st place votes

The Sierpinski Gasket (Recursive Construction)

- **Start.** Start with any solid triangle *ABC* (a). (Often an equilateral triangle or a right triangle is used, but here we chose a random triangle to underscore the fact that it can be a triangle of arbitrary shape.)

- **Step 1.** Remove the triangle connecting the midpoints of the sides of the solid triangle. This give the shape shown in (b)– consisting of three solid triangles each a half-scale version of the original and a hole where the middle triangle used to be.

- **Step 2.** To each of the three triangles in (b) apply *Procedure SG*. The result is the "gasket" shown in (c) consisting of $3^2 = 9$ triangle each at one-fourth the scale of the original triangle, plus three small holes of the same size and one larger hole in the middle.

- **Step 3.** To each of the three nine triangles in (c) apply *Procedure SG*. The result is the "gasket" shown in (d) consisting of $3^3 = 27$ triangle each at one-eighth the scale of the original triangle, nine small holes of the same size, three medium-size holes and one large hole in the middle.

- **Step 4, 5, etc.** Apply *Procedure SG* to each triangle in the "gasket" obtained in the previous step.

You can think of the figure on the right as a picture of the Sierpinski gasket (in reality it is the gasket obtained at step 7 of the construction process.

The Sierpinski gasket is clearly a fairly complicated geometric shape, and yet it can be defined in two lines using the following **recursive** replacement rule.

The Sierpinski Gasket

- **Start:** Start with an arbitrary solid triangle Δ .
- **Replacement rule:** Whenever you see a Δ apply *Procedure SG* to it.

As a geometric object existing in the plane, the Sierpinski gasket should have an area, but it turns out that its area is infinitely small, smaller than any positive quantity. Therefore,

- **The Sierpinski gasket has zero area but infinitely long boundary.**

The Chaos Game

This example involve the laws of chance. We start with an arbitrary triangle with vertices A, B, and C and an honest die (a). Before we start we assign two of the six possible outcomes of rolling the die to each of the vertices of the triangle.

- **Start.** Roll the die. Start at the "winning" vertex. Say we roll a 5. We then start at vertex C at figure (b).

- **Step 1.** Roll the die again. Say we roll a 2, so the winner is vertex A. We now *move to the point M^1 halfway between the previous position C and the winning vertex A*. Mark a point at the new position M^1 (see figure c).

- **Step 2.** Roll the die again, and *move to the point M^1 and the winning vertex*. [Say we roll a 3– the move then is to M^2 halfway between M^1 and B as shown in(d).] Mark a point at the new position $M^{2..}$

- **Step 3, 4, etc.** Continue rolling the die, each time *moving halfway between the last position and the winning vertex* and marking that point.

The Twisted Sierpinski Gasket

Our next example is a simple variation of the original Sierpinski gasket. We will call it the *twisted Sierpinski gasket*. For convenience, we will use the term *Procedure TSG* to describe the combination of the two moves ("cut" and then "twist").

- **Cut.** Cut the middle out of a triangle (b).
- **Twist.** Translate each of the midpoints of the sides by a small random amount and in a random direction (c).

When we repeat procedure TSG in an infinite **recursive** process, we get the twisted Sierpinski gasket.

The Twisted Sierpinski Gasket (Recursive Construction)

- **Start.** Star with an arbitrary solid triangle such as shown in figure (a).

- **Step 1.** Apply *Procedure TSG* to the starting triangle. This gives the "twisted gasket" shown in (b), with three twisted triangles and a (twisted) hole in the middle.

- **Step 2.** To each of the three triangles in (b), apply *Procedure TSG*. The result is the "twisted gasket" shown in (c), consisting of nine twisted triangles and four holes of various sizes.

- **Step 3, 4, etc.** Apply *Procedure TSG* to each triangle in the "twisted gasket" obtained in the previous step.

The figure above shows an example of a twisted Sierpinski gasket at step 7 of the construction. Even without touch-up, we can see that this image has the unmistakable look of a mountain.

The construction of the twisted Sierpinski gasket can be also described by a two-line **recursive** replacement rule.

Twisted Sierpinski Gasket

- **Start:** Start with an arbitrary solid triangle.
- **Replacement rule:** Wherever you see a solid triangle, apply Procedure TSG to it.

The Mandelbrot Set

Complex Numbers and Mandelbrot Sequences

The Mandelbrot set can be described mathematically b a **recursive** process involving simple computations with complex numbers. The complex number $(a + bi)$ can be identified with the point (a,b) in a Cartesian coordinate system as shown to the right.

Mandelbrot Sequence

The key concept in the construction of the Mandelbrot set is that of a *Mandelbrot sequence*. A **Mandelbrot sequence** (with seed *s*) is an infinite sequence of complex numbers that starts with an arbitrary complex number *s* and then each successive term in the sequence is obtained recursively by *adding the seed s to the previous term squared*.

Much like the Koch snowflake and the Sierpinski gasket, a Mandelbrot sequence can be defined by means of a **recursive** replacement rule:

Mandelbrot Sequence

- **Start:** Choose an arbitrary complex number *s*, called the **seed** of the Mandelbrot sequence. Set the seed *s* to be the initial term of the sequence ($s^0 = s$).
- **Procedure M:** To find the next term in the sequence, square the preceding term and add the seed ($s^{N+1} = s^{2N} + s$).

The Mandelbrot Set

If the Madelbrot sequence is *periodic or attracted*, the seed is a point of the Mandelbrot set and assigned the color black; if the Mandelbrot sequence is *escaping*, the seed is a point outside the Mandelbrot and assigned color that depends on the speed at which the sequence is escaping (hot colors for slowly escaping sequences, cool colors for fast escaping sequences.

Perfect Right Triangles

Right triangles with sides that are whole numbers are called perfect right triangles. 3/4/5, 5/12/13 and 7/24/25 triangles are examples of perfect right triangles. A 3/4/5 perfect right triangle can be found in the "King's Chamber" of the Great Pyramid in Egypt. The Pythagorean Theorem is used to measure the sides of right triangles.
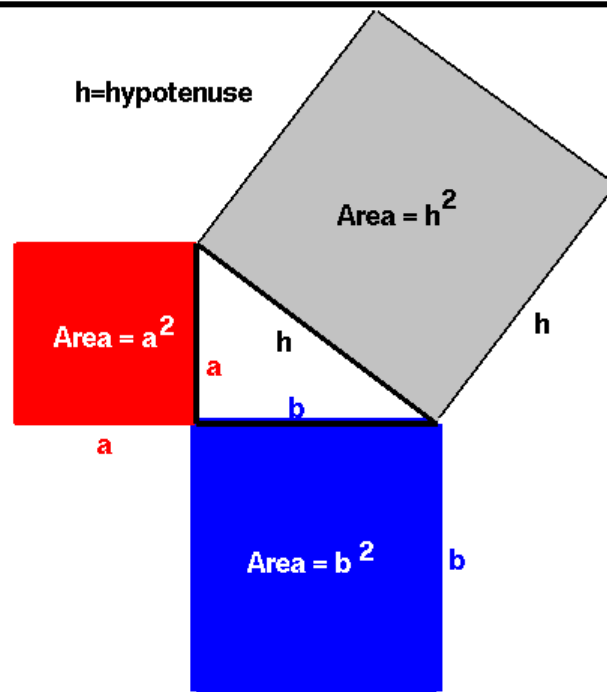

Pythagorean Theorem — NASA Glenn Research Center

For any right triangle with sides a and b and hypotenuse h, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

h=hypotenuse

Area = a^2

Area = h^2

Area = b^2

$$h^2 = a^2 + b^2$$

To better understand certain problems involving aircraft and propulsion it is necessary to use some mathematical ideas from [trigonometry](#) the study of triangles. Let us begin with some definitions and terminology which we will use on this slide. We start with a **right triangle**. A right triangle is a three sided figure with one angle equal to 90 degrees. A 90 degree angle is called a **right angle** and that is where the right triangle gets its name. We define the side of the triangle opposite from the right angle to be the **hypotenuse, h**. It is the longest side of the three sides of the right triangle. *The word "hypotenuse" comes from two Greek words meaning "to stretch", since this is the longest side.* We are going to label the other two sides **a** and **b**. The **Pythagorean Theorem** is a statement relating the lengths of the sides of **any** right triangle.

The theorem states that:

*For any right triangle, the square of the hypotenuse
is equal to the sum of the squares of the other two sides.*

Mathematically, this is written:

$$h^2 = a^2 + b^2$$

The theorem has been known in many cultures, by many names, for many years. Pythagoras, for whom the theorem is named, lived in ancient Greece, 2500 years ago. It is believed that he learned the theorem during his studies in Egypt. The Egyptians probably knew of the relationship for a thousand years before Pythagoras. The Egyptians knew of this relationship for a triangle with sides in the ratio of "3 - 4 - 5".

$$5^2 = 3^2 + 4^2$$

$$25 = 9 + 16$$

Pythagoras generalized the result to any right triangle. There are many different algebraic and geometric **proofs** of the theorem. Most of these begin with a construction of squares on a sketch of a basic right triangle. On the figure at the top of this page, we show squares drawn on the three sides of the triangle. A square is the special case of a rectangle in which all the sides are equal in length. The [area](#) **A** of a rectangle is the product of the sides. So for a square with a side equal to **a**, the area is given by:

$$A = a * a = a^2$$

So the Pythagorean theorem states the area **h^2** of the square drawn on the hypotenuse is equal to the area **a^2** of the square drawn on side **a** plus the area **b^2** of the square drawn on side **b**.

Here's an interactive Java program that let's you see that this area relationship is true:

This page shows an interactive Java applet which demonstrates the Pythagorean Theorem.

We begin with a right triangle on which we have constructed squares on the two sides, one red and one blue. We are going to break up the pieces of these two squares and move them into the grey square area on the hypotenuse. We won't loose any material during the operation. So if we can exactly fill up the square on the hypotenuse, we have shown that the areas are equal. You work through the construction by clicking on the button labeled "Next". You can go "Back" and repeat a section, or go all the way back tothe beginning by clicking on "Reset".

What is it doing? The first step rotates the triangle down onto the blue square. This cuts the blue square into three pieces, two triangles and a red rectangle. The two triangles are exactly the same size as the original triangle. The "bottom" of the original triangle exactly fits the vertical side of the square, because the sides of a square are equal. The red rectangle has its vertical sides equal to the base of the original triangle, and its horizontal sides equal to the difference between the "bottom" side and the "vertical" side of the original triangle. Using the terminology from the figure at the top of this page, the dimensions of the red rectangle are:

vertical length = b

horizontal length = b - a

The next step is to move the red rectangle over adjacent to the red square. The rectangle sticks out the top of the red square and the two triangles remain in the blue square. The next step is to move one of the blue triangles vertically into the hypotenuse square. It fits exactly along the side of the hypotenuse square because the sides of a square are equal. The next step is to move the other blue triangle into the hypotenuse square. (We are half way there!) The next step is to slide the form of the original triangle to the left into the red region. The triangle cuts the red region into three pieces, two triangles and a small yellow square. The original triangle fits exactly into this region because of two reasons; the vertical sides are identical, and the horizontal side of the red region is equal to the length of the red square plus the horizontal length of the red rectangle which we moved. The horizontal length of the red region is:

horizontal length = a + (b - a) = b

The horizontal length of the red region is exactly the length of the horizontal side of the original triangle. The yellow square has dimensions **b - a** on each side. The next step is to move one of the red triangles into the hypotenuse square. Again it's a perfect fit. The next step is to move the final red triangle into the hypotenuse square. Now if we look at the grey square that remains in the hypotenuse square, we see that its dimensions are **b - a**; the long side of the triangle minus the short side. The final step is to move the yellow square into this hole. It's a perfect fit and we have used all the material from the original red and blue squares.

Platonic Solids

A Platonic solid is a convex polyhedron. Platonic solids are made up of equal faces and are made up of congruent regular polygons. There are 5 Platonic solids. They are named for the number of faces: tetrahedron - 4 faces, hexahedron - 6 faces, octahedron - 8 faces, dodecahedron - 12 faces, and icosahedron - 20 faces. The ancient Greeks believed that these 5 Platonic solids symbolized the elements, with the dodecahedron symbolizing the heavens.

The Platonic Solids belong to the group of geometric figures called *polyhedra.*

A **polyhedron** is a solid bounded by plane polygons. The polygons are called **faces**; they intersect in **edges**, the points where three or more edges intersect are called **vertices**.

A **regular** polyhedron is one whose faces are identical regular polygons. Only five **regular solids** are possible

cube tetrahedron octahedron icosahedron dodecahedron

These have come to be known as **the Platonic Solids**

Plato associates four of the *Platonic Solid* with the four elements. He writes,

We must proceed to distribute the figures [the solids] we have just described between fire, earth, water, and air. . .

Let us assign the cube to earth, for it is the most immobile of the four bodies and most retentive of shape

the least mobile of the remaining figures (icosahedron) to water
the most mobile (tetrahedron) to fire
the intermediate (octahedron) to air

Note that earth is associated with the cube, with its six square faces. This lent support to the notion of the foursquaredness of the earth.

Archimedean Solids

Archimedean solids are made up of two or more different regular polygons. There are 13 different solids. 7 of the 13 solids can be made by truncating a platonic solid.

Below is the Greek text and an English translation from the fifth book of the "[Synagoge" or "Collection](#)" of the Greek mathematician [Pappus of Alexandria](#), who lived in the beginning of the fourth century AD. This book gives the first known mention of the thirteen "Archimedean solids", which Pappus lists and attributes to Archimedes. However, Archimedes makes no mention of these solids in any of his extant works.

The earliest surviving manuscript of Pappus's "Collection" is located in the [Vatican Library](#) and dates from the tenth century (Codex Vaticanus Graecus 218). A photograph of a pair of pages from this manuscript can be downloaded from a [Web site of the Library of Congress Vatican Exhibit](#).

[Johannes Kepler](#) (1571-1630) was the next to write about the Archimedean solids collectively in his book [Harmonices Mundi](#), although some of the solids were separately rediscovered and discussed by others. Kepler sharpened Pappus's somewhat loose definition of the solids and gave a proof that there are precisely thirteen of them (Book II, "De Congruentia Figurarum Harmonicarum", Proposition XXVIII, [pages 61-65](#)). He also provided the first known illustration of them as a set ([see pages 62 & 64](#)) and gave them their modern names, which are reproduced below. Other representations and properties of these solids can be found at [Wikipedia](#) and a [site maintained by Tom Gettys](#).

Although many solid figures having all kinds of surfaces can be conceived, those which appear to be regularly formed are most deserving of attention. Those include not only the five figures found in the godlike Plato, that is, the tetrahedron and the cube, the octahedron and the dodecahedron, and fifthly the icosahedron, but also the solids, thirteen in number, which were discovered by Archimedes and are contained by equilateral and equiangular, but not similar, polygons.

Πολλὰ γὰρ ἐπινοῆσαι δυνατὸν στερεὰ σχήματα παντοίας ἐπιφανείας ἔχοντα, μᾶλλον δ᾽ ἄν τις ἀξιώσειε λόγον τὰ τετάχθαι δοκοῦντα. Ταῦτα δ᾽ ἐστὶν οὐ μόνον τὰ παρὰ τῷ θειοτάτῳ Πλάτωνι πέντε σχήματα, τουτέστιν τετράεδρόν τε καὶ ἑξάεδρον, ὀκτάεδρόν τε καὶ δωδεκάεδρον, πέμπτον δ᾽ εἰκοσάεδρον, ἀλλὰ καὶ τὰ ὑπὸ Ἀρχιμήδους εὑρεθέντα τρισκαίδεκα τὸν ἀριθμὸν ὑπὸ ἰσοπλεύρων μὲν καὶ ἰσογωνίων οὐχ ὁμοίων δέ πολυγώνων περιεχόμενα.

Truncated Tetrahedron

The first is a figure of eight bases, being contained by four triangles and four hexagons.

Τὸ μὲν γὰρ πρῶτον ὀκτάεδρόν ἐστιν περιεχόμενον ὑπὸ τριγώνων δ΄ καὶ ἑξαγώνων δ΄.

Cuboctahedron

After this come three figures of fourteen bases, the first contained by eight triangles and six squares,

Τρία δὲ μετὰ τοῦτο τεσσαρεσκαιδεκάεδρα, ὧν τὸ μὲν πρῶτον περιέχεται τριγώνοις η΄ καὶ τετραγώνοις ς᾽,

Truncated Octahedron

the second by six squares and eight hexagons,

τὸ δὲ δεύτερον τετραγώνοις ς΄ καὶ ἑξαγώνοις η΄,

Truncated Cube

and the third by eight triangles and six octagons.

τὸ δὲ τρίτον τριγώνοις η΄ καὶ ὀκταγώνοις ς΄.

Rhombicuboctahedron

After these come two figures of twenty-six bases, the first contained by eight triangles and eighteen squares,

Μετὰ δὲ ταῦτα ἐκκαιεικοσάεδρά ἐστιν δύο, ὧν τὸ μὲν πρῶτον περιέχεται τριγώνοις η΄ καὶ τετραγώνοις ιη΄,

Truncated Cuboctahedron

the second by twelve squares, eight hexagons and six octagons.

τὸ δὲ δεύτερον τετραγώνοις ιβ΄, ἑξαγώνοις η΄ καὶ ὀκταγώνοις ς΄.

Icosidodecahedron

After these come three figures of thirty-two bases, the first contained by twenty triangles and twelve pentagons,

Μετὰ δὲ ταῦτα δυοκαιτριακοντάεδρά ἐστιν τρία, ὧν τὸ μὲν πρῶτον περιέχεται τριγώνοις κ΄ καὶ πενταγώνοις ιβ΄,

Truncated Icosahedron

the second by twelve pentagons and twenty hexagons,

τὸ δὲ δεύτερον πενταγώνοις ιβ΄ καὶ ἑξαγώνοις κ΄,

Truncated Dodecahedron

and the third by twenty triangles and twelve decagons.

τὸ δὲ τρίτον τριγώνοις κ΄ καὶ δεκαγώνοις ιβ΄.

Snub Cube

After these comes one figure of thirty-eight bases, being contained by thirty-two triangles and six squares

Μετὰ δὲ ταῦτα ἕν ἐστιν ὀκτωκαιτριακοντάεδρον περιεχόμενον ὑπὸ τριγώνων λβ΄ καὶ τετραγώνων ς΄.

Rhombicosidodecahedron

After this come two figures of sixty-two bases, the first contained by twenty triangles, thirty squares and twelve pentagons,

Μετὰ δὲ τοῦτο δυοκαιεξηκοντάεδρά ἐστι δύο, ὧν τὸ μὲν πρῶτον περιέχεται τριγώνοις κ΄ καὶ τετραγώνοις λ΄ καὶ πενταγώνοις ιβ΄,

the second by thirty squares, twenty hexagons and twelve decagons.

τὸ δὲ δεύτερον τετραγώνοις λ΄ καὶ ἑξαγώνοις κ΄ καὶ δεκαγώνοις ιβ΄.

Truncated
Icosidodecahedron

Snub Dodecahedron

After these there comes lastly a figure of ninety-two bases, which is contained by eighty triangles and twelve pentagons.

Μετὰ δὲ ταῦτα τελευταῖόν ἐστιν δυοκαιενενηκοντάεδρον, ὃ περιέχεται τριγώνοις π΄ καὶ πενταγώνοις ιβ΄.

Stellations of The Platonic & Archimedean Solids

When a Platonic or Archimedean solid is stellated they create new forms. The process of stellation creates a 3D form with tetrahedrons, or pyramids. For example, if you stellate a cube, a cube based pyramid will be created. Stellation can create a large number of new forms.
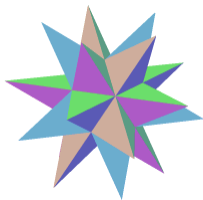
Links for stellations of Platonic solids

Stellations of the Dodecahedron Java applet that shows various stellations rotating and morphing into each other. Very cool.
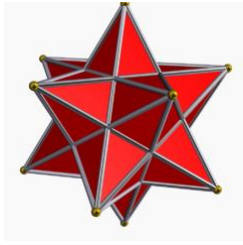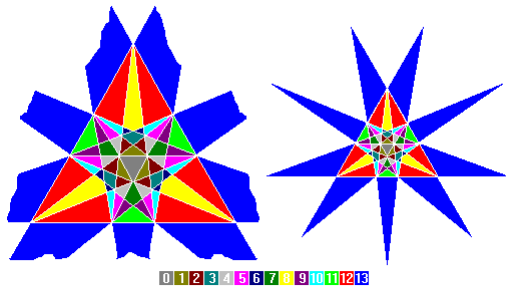
Small Stellated Dodecahedron
MathWorld article

Great Stellated Dodecahedron MathWorld article

[Small Stellated Dodecahedron](#) Wikipedia article

[Stellated Icosahedra](#)There are 59 stellations of the icosahedron. These pages contain various images of these stellations and some background information.

[Stellations of the Icosahedron](#) Informative article in [Symmetry, Crystals and Polyhedra](#) by Steven Dutch

[First stellation of icosahedron](#)Wikipedia article

80 Stellated dodecahedron I

81 Stellated icosahedron I

82 Stellated dodecahedron II

83 Stellated icosahedron II

84 Stellated icosahedron III


85 Stellated Octahedron

To understand the stellated polyhedra, you need to look under the surface. The faces of these polyhedra are not the external facets, but rather larger polygons that extend through the middle of the figure, intersecting each other. This is easiest to see with the stellated octahedron (85). The large triangular faces cut through each other, forming two intersecting tetrahedra. One can obtain this figure by starting with an octahedron (which would be visible from the inside of this model), and allowing the faces to expand symmetrically until they intersect each other in the edges of a new polyhedron. This process is called stellation. Performing this process of stellation on a dodecahedron yields the small stellated dodecahedron (80), whose faces are pentagrams (five-pointed stars). Allowing these faces to expand even further (stellating the stellation) yields the great dodecahedron (not shown), and stellating this figure in turn yields the great stellated dodecahedron (82). Continuing the process yields nothing new, thus the dodecahedron has three stellations. The icosahedron has 59 stellations, of which three are shown here: 81, 83, and 84. Four of the stellated polyhedra are regular. These are the Kepler-Poinsot polyhedra: the small stellated dodecahedron; its dual, the great dodecahedron; the great stellated dodecahedron; and its dual, the great icosahedron.

Metatron's Cube

Metatron is the name of the angel that guards God's throne in Judaism. The figure of Metatron's Cube has been in sacred art for thousands of years. The 5 Platonic solids can be found within the cube. Because it contains the 5 Platonic solids, it is thought that it contains the building blocks of creation.

One of the key concepts of sacred geometry is the [flower of life](#). Within this figure, through various expansions, contractions, and connections, one can find a blueprint for concepts espoused by Platonic philosophy and even ancient [Judaism](#) to be the basis for the universe and life itself (Frissell, Something

197). When considering the flower of life shape, it is important to realize that this is actually a two-dimensional representation of a three-dimensional form. Therefore, these two figures actually represent the same shape:
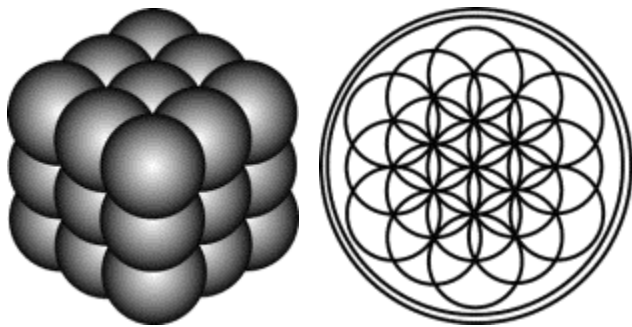

(Alloca 1)

   In the figure on the left, only nineteen of the spheres are visible, but by representing the shape two-dimensionally, one can see that in actuality, there are a total of twenty-seven spheres.

   Through expanding the flower of life shape to contain 125 spheres, one creates a new shape, from which the �fruit of life� can be extracted (from this vantage point, one sees only thirteen spheres in the fruit of life, but in actuality, there are seventeen, with two both in front and behind of the middle sphere). By connecting the middle point of each sphere to the middle point of all others, Metatron�s cube can be found (Allocca 1).

Flower of Life Stage   Fruit of Life Stage   Fruit of Life   Metatron's Cube
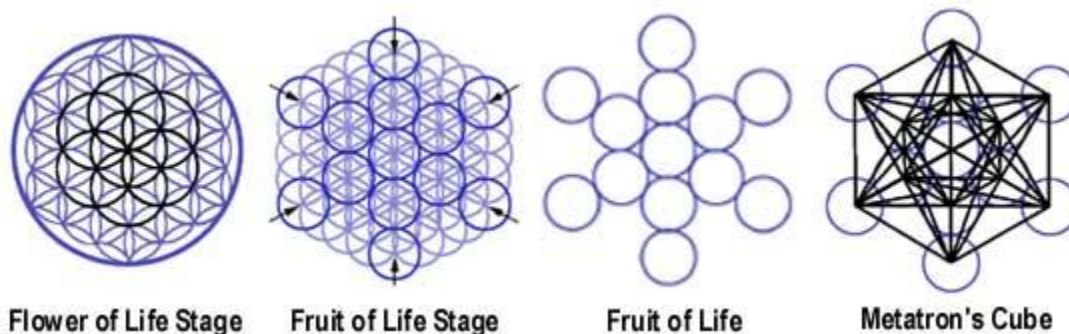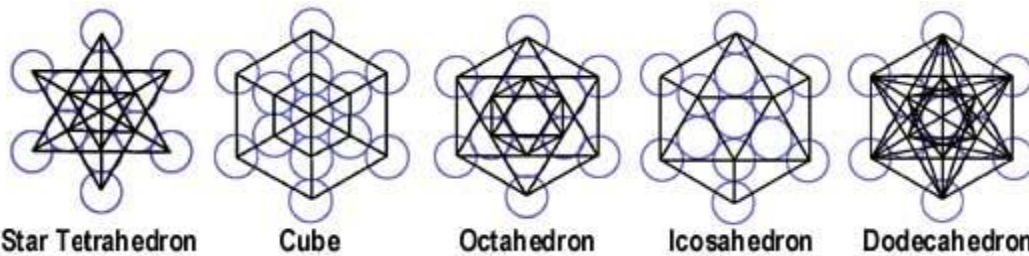
(Melchizidek 7)
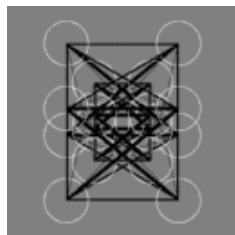
 (Three-Dimensional View of Metatron's Cube)

Although the shape of Metatron's cube has it's own significance in relation to sacred geometry and other forms of philosophy, it is more widely recognized as the basis for deriving the platonic solids, as seen here:

| Star Tetrahedron | Cube | Octahedron | Icosahedron | Dodecahedron |

(Melchizidek 8)

The platonic solids are a group of five three-dimensional solid shapes, each containing all congruent angles and sides. Also, if circumscribed with a sphere, all vertexes would touch the edge of the sphere. It was Euclid who would later prove in his book, entitled The Elements, that these are the only five shapes which actually fit this criteria (Weisstein 1).
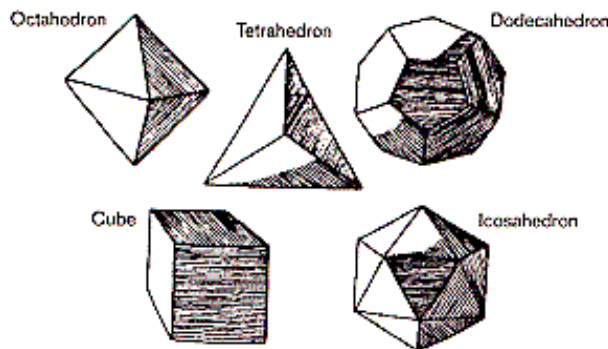
(Frissell, Something 200)

In his book, Timaeus, written in approximately 350 BC, Plato first described these solids, linking them to different elements of reality. The tetrahedron, containing four sides, and actually found twice within Metatron�s cube (the star tetrahedron is a combination of two tetrahedrons), is used to represent fire. The cube, containing six sides, and also found twice within Metatron�s cube, represents the earth. The octahedron, containing eight sides, represents the air. The icosahedron, containing twenty sides, represents the water. Finally, the dodecahedron, containing twelve sides, is used to represent the cosmos (Weisstein 1).

The concept of the entire universe being made up of four basic elements (earth, fire, water, and air) dates back over a hundred years earlier than Plato�s Timaeus, with the work of the Greek philosopher Empedocles, who lived from approximately 493 to 433 BC. He theorized that all matter is made merely of varying combinations and proportions of these elements. It is possible, however, that this concept also existed long before Empedocles ever wrote about it. He was a disciple of Pythagoras, who had also been greatly influenced by the ancient Egyptians, and Empedocles could have, in theory, gotten his information handed down from either of these sources.

However, it was Plato, who was born six years after Empedocles� death, who would apply a logical formula to assigning the elements to the Platonic Solids: �Let us assign the cube to earth, for it is the most immobile of the four bodies and most retentive of shape�the least mobile of the remaining figures (icosahedron) to water, the most mobile (tetrahedron) to fire, the intermediate (octahedron) to air� However, this still leaves the dodecahedron, which, according to Plato, �the god used for embroidering the constellations on the whole heaven" (Calter 2-6). In this way, the Platonic Solids can be used to represent the entire universe. Other shapes within sacred geometry continue to expand

upon these principles and provide valuable insight into the [nature of all things, from mankind to the cosmos](.).

The most difficult part of this project was narrowing such a broad original topic like sacred geometry into a more specific subject such as Metatron�s Cube and the Platonic Solids. However, I am glad that I was able to do a large amount of general research before specifying because this way of doing things allowed me to increase my own knowledge on the subject of sacred geometry. I would definitely like to continue research on the subject in many other areas. Specifically, in my research I found some information linking parts of sacred geometry to ancient Jewish beliefs, and I would like to find out more about this connection. Although it may have seemed a daunting task at first, to gain a complete understanding of sacred geometry, the realization that I still have so much to learn is comforting in that this interesting topic will be able to stay with me for a long time.
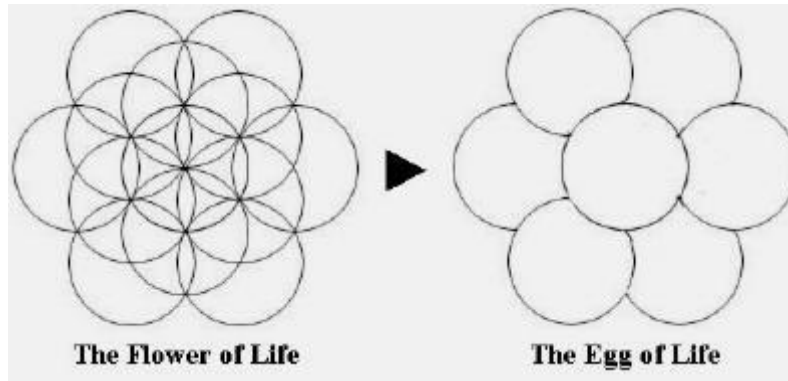
The Flower of Life

Images of the Flower of Life have been found all around the world and in most ancient civilizations. The Platonic solids, Metatron's Cube, the Vesica Piscis can be found within the Flower of Life. Other sacred geometric forms such as the Seed of Life, the Tripod of Life, the Egg of Life, the Fruit of Life and the Tree of Life are also found inside the Flower if Life. This sacred shape is said to contain patterns of creation.

The "Flower of Life" can be found in all major religions of the world. It contains the *patterns of creation* as they emerged from the "Great Void". Everything is made from the Creator's thought. After the creation of the *Seed of Life* the same vortex's motion was continued, creating the next structure known as the *Egg of Life.*

This structure forms the basis for music, as the distances between the spheres is identical to the distances between the tones and the half tones in music. It is also identical to the cellular structure of the third embryonic division (The first cell divides into two cells, then to four cells then to eight). Thus this same structure as it is further developed, creates the human body and all of the energy systems including the ones used to create the *Merkaba*. If we continue creating more and more spheres we will end up with the structure called the *Flower of Life.*
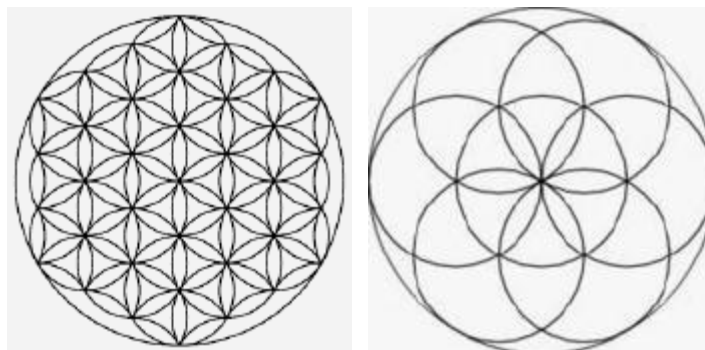
The Flower of Life (left) and the Seed of Life (right).

The Flower of Life ▶ **The Egg of Life**

The flower of life holds a secret symbol created by drawing 13 circles out of the Flower of Life. By doing this, one can discover the most important and sacred pattern in the universe. This is the source of all that exists; it's called the *Fruit of Life*. It contains 13 informational systems. Each one explains another aspect of reality. Thus these systems are able to give us access to everything ranging from the human body to the galaxies. In the first system, for example, it's possible to create any molecular structure and any living cellular structure that exists in the universe. In short every living creature.

The most common form of the "Flower of Life" is hexagonal pattern (where the center of each circle is on the circumference of six surrounding circles of the same diameter), made up of 19 complete circles and 36 partial circular arcs, enclosed by a large circle.

The "Seed of Life" is formed from seven circles being placed with sixfold symmetry, forming a pattern of circles and lenses, which acts as a basic component of the Flower of Life's design.

Sources

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- [The Point](): A webpage with an illustration of the point.
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- [The Golden Ratio](): A webpage discussing the golden ratio.
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- [The Vesica Piscis](): A webpage with information about and illustrations of the vesica piscis.
- [Equiangular Spirals](): A webpage with information about and illustrations of equiangular spirals.
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